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Community cohesion and assimilation equilibria

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That analysis drew on two building blocks. First, from the perspective of the theory of social proximity and group affiliation (as per Akerlof, 1997), the assimilation of a migrant was understood to reduce his social distance from the native population. Second, noting the large amount of evidence that income comparisons influence people's wellbeing, this influence was quantified by means of a measure of relative deprivation, which in turn was incorporated in a utility function that is additive in income, cost of assimilation effort, and the measure of relative deprivation.[3] Fan and Stark inquired how closely migrants choose to align themselves with the native inhabitants (henceforth referred to as natives), who, being more productive and wealthier than the migrants, expose the migrants to relative deprivation. The equilibrium assimilation level of the migrants was shown to be lower than the level that would have been chosen had the utility function not incorporated a relative deprivation component.[4]

In this paper, we expand that analysis and, in addition, we expand the unit of analysis, beyond that starting point, addressing the question why are there stark differences in the extent of assimilation of different communities of migrants. Divergence is evident across different ethnic groups of migrants even in the same host country (Gordon, 1964; Alba and Logan, 1993; Iceland and Nelson, 2008); across migrants with different levels of education (Gijsberts and Vervoort, 2009; van Tubergen and Kalmijn, 2009); and across different generations of migrants from a given country of origin (Gans, 1992; Portes and Zhou, 1993; Perlmann and Waldinger, 1997, for migrants in the US; and the references provided in Thomson and Crul, 2007, for migrants in European countries). The variation between "total acculturation" and "rigid segregation" (Alba and Nee, 2003) has been particularly well documented in the case of migrants to the US (Massey and Denton, 1987; Kroneberg, 2008; Telles and Ortiz, 2008; Jiménez, 2010).

Consider the link with evidence that migrants who live in highly concentrated urban communities (that is, in communities with a great many fellow migrants) do not assimilate much; for example, their proficiency in the host country's language stays low, which in turn has a negative effect on their earnings (McManus et al., 1983; Tainer, 1988; Shields and Price, 2002; Chiswick and Miller, 2002, 2005; Cutler et al., 2008). Then, migrants' segregation and lowered incomes can cement into a "culture of poverty" (Wilson, 1987). A concentration of poor migrants can have adverse effect on the urban native inhabitants; for example, an increase in poor people in a central city location can cause an outflow of richer native inhabitants and deterioration of the center area (Kanemoto, 1980). A different effect is identified by Ottaviano and Peri (2005). Drawing on US census data for 1970–1990, they report that the productivity of US-born workers was higher in cities with richer linguistic diversity, and that the presence of assimilated non-natives (who speak English and who have been in the US for a long time) had the most beneficial effect on the productivity of US-born workers. Especially when the extent of the assimilation of migrants bears meaningfully on the wellbeing of the native inhabitants, policy makers will want to understand what governs assimilation behavior.

Usually, migrants are not compelled to live in high-concentration areas; rather, they choose to (Bartel, 1989; Dunlevy, 1991; Bauer et al., 2005). We maintain that "fear" of social proximity to the native inhabitants causes migrants to live in (or move into) neighborhoods with large concentrations of migrants, thereby increasing their concentration; the choice of geographical space springs from preferences with regard to social space. Migrants live in concentrations because of their fear of assimilation or failure to assimilate, rather than fail to assimilate because they live in concentrations of migrants. Our view is not that concentration explains non-assimilation, but rather that non-assimilation explains concentration: migrants elect not to assimilate and consequently they concentrate. Whereas the line of reasoning of the conventional approach is that a low level of assimilation is the result of living in concentrations (Chiswick and Miller, 1995; Lazear, 1999, 2005), our approach is that both a low level of assimilation *and* concentrated living are the result of a reluctance to assimilate. This perspective is not based on the notion that patterns of concentration reflect diversity in (an exogenous) ability to assimilate, with low-ability migrants choosing high concentrations; rather, the intensity of assimilation and the intensity of concentration are both taken as matters of choice.

Recent research on assimilation, concentration, and segregation recognizes the importance of social and cultural considerations. For example, Verdier and Zenou (2017) who study assimilation, employ the concept of networks as a representation of social space; the outcomes of their modeling depend on the shape (density) of the network, and on the cost of expending effort to assimilate. Bezin and Moizeau (2017), who present a model of neighborhood formation in the context of cultural dynamics, link ethnic urban segregation with a preference for the preservation of certain cultural traits. However, these studies do not consider distaste of social proximity as a determinant of assimilation.

In order to explain why different communities of migrants exhibit different degrees of assimilation, we study the inner workings of the communities, asking how the characteristics of a community of migrants influence the community's equilibrium level of assimilation. Rather like in evolutionary biology, we "stress test" the prevailing equilibrium when a "mutant" migrant appears. The mutation takes the form of a migrant who has an enhanced ability to assimilate, brought about exogenously. The superior ability is expressed as a cost of assimilation that is lower than that of the other migrants. Henceforth we refer to this migrant as a mutant migrant.

We find that if undeterred, a mutant migrant will assimilate more than the other members of his community. When the mutant migrant acts on his enhanced ability to assimilate without impediment and obtains higher earnings, he exposes the other migrants in his community to more relative deprivation. The community will therefore have an incentive to safeguard the prevailing assimilation equilibrium and dissuade the mutant member from acting on his enhanced ability. The community's success in preserving the prevailing equilibrium depends on its ability to impose a sanction on the mutant so as to discourage him from acting on his enhanced ability; we refer to this ability as cohesion. The community's sanction takes the form of shunning, namely curtailing its affinity with, the mutant member. The sanction will harm the mutant member because it will push him further "into the arms" of the native population, increasing his proximity to the natives, which will exacerbate his relative deprivation.

Our model tracks the stability of the pre-mutation assimilation equilibrium as a function of the strength of the sanction / the degree of cohesion of the community. A tightly knit migrant community is able to impose an effective sanction to discourage a mutant member from acting on his enhanced ability to assimilate. Such a community can preserve the stability of the assimilation equilibrium. On the other hand, a loose-knit community will not be able to marshal the discipline and level of enforcement of a sanction that will render its sanction powerful enough to discourage the mutant from deviating. Unimpeded, the mutant member will then act on his enhanced ability to assimilate. But then, in response to the relative deprivation inflicted by the mutant's behavior on the "normal" members, these members will follow in his

[3] The idea that relative income influences the individual's wellbeing dates back at least to Veblen (1899), who has shown that higher earnings of others can depress one's utility. Becker (1974) and Yitzhaki (1979) lay down theoretical foundations of a relative deprivation approach to comparisons between individuals. Recent empirical studies have demonstrated the importance of relative deprivation: Walker and Smith (2002), Eibner and Evans (2005), Luttmer (2005), and Clark et al. (2008). Cole et al. (1992, 1998), and Postlewaite (1998) explore the microeconomic foundations of the role of relative income in the determination of individuals' welfare.

[4] The distaste for relative deprivation is not the only possible explanation for non-assimilation. For example, for migrants who derive utility from interacting with others who share the same culture or speak the same language, non-assimilation has a consumption value even if it reduces labor productivity. However, we do not consider this specific line of reasoning particularly revealing because, in and of itself, it is subsumed by our argument: as shown in subsequent sections, it is the fear of loss of this value that renders sanctions against a deviant migrant effective.

footsteps: in order to reduce their loss of utility from experiencing increased relative deprivation from a comparison with the mutant, the "normal" members will assimilate more. Thus, the stability of the equilibrium will be disrupted.

From these considerations we conclude that the extent of assimilation of a community of migrants is inversely related to the community's cohesion: in the presence of a mutation, a less tightly knit community will assimilate more. The community's cohesion is key to the community's immunity to adverse relative deprivation consequences that would be inflicted on it if a mutant appears. It is this cohesion that determines the stability of the community's equilibrium level of assimilation.

2. Assimilation as a game: Introduction

We consider a city populated by two types of individuals: natives and migrants. The natives constitute the "mainstream culture," and are richer than the migrants. To concentrate on essentials, we assume that the incomes of the natives are constant and exogenous to the model.

Each of $n \geq 2$ migrants decides how much effort to exert in order to assimilate into the mainstream culture. If a migrant is better assimilated into the mainstream culture, he will earn a higher income; assimilation leads to the acquisition of host city specific human capital. However, assimilating more entails closer social proximity to the natives when making personal comparisons. We assume that personal comparisons matter to a migrant, and that he is concerned about adverse outcomes of such comparisons. We refer to this concern as sensing or experiencing relative deprivation. A decision by a migrant to exert an income-boosting higher level of effort can negatively affect other migrants who, when comparing themselves to him, will feel relatively deprived. This perspective invites modeling assimilation behavior as a game between the migrants. In this game, the utility / payoff function of migrant i, $i \in \{1, 2, \ldots, n\}$, is

$$u_i(x_i, \mathbf{x}_{-i}) = Y(x_i) - RD(x_i, \mathbf{x}_{-i}) - c_i C(x_i), \tag{1}$$

where $x_i \in [0, 1]$ is the effort exerted by migrant i to assimilate (this effort converts into host city specific human capital); $Y(\cdot)$ is migrant i's income; $RD(\cdot, \cdot)$ is migrant i's relative deprivation; $C(\cdot)$ is the cost of exerting assimilation effort; $c_i \in (0, 1]$ is an individual parameter representing the potential reduction in i's cost of assimilation (a migrant's capacity to assimilate); and $\mathbf{x}_{-i}$ is the vector of the levels of the assimilation effort chosen by the other migrants: $\mathbf{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$.

We make the following additional assumptions.

$$Y'(x_i) > 0 \text{ and } Y''(x_i) < 0 \text{ for } 0 < x_i < 1; \ \lim_{x_i \to 0} Y'(x_i) = \infty, \tag{2}$$

and

$$C'(x_i) > 0 \text{ and } C''(x_i) > 0 \text{ for } 0 < x_i < 1; \ \lim_{x_i \to 1} C'(x_i) = \infty. \tag{3}$$

To define $RD(\cdot, \cdot)$, we introduce a function $F[A - Y(x_i)]$, where A is the average income of the comparison group of migrant i, such that $F(v)$ is differentiable for every $v \in \mathbf{R}$, and

$$F(v) \equiv 0 \text{ for } v \leq 0; F'(v) > 0 \text{ for } v > 0; \text{ and } F''(v) \geq 0. \tag{4}$$

From the assumptions in (2) and (4) it follows that as a function of x_i, $F[A - Y(x_i)]$ is convex.[5] The function $F(\cdot)$ encompasses the ideas that a migrant experiences disutility when other members of his comparison group(s) earn more than he does, and that the extent of this disutility rises with the difference between the average income of the comparison group and the migrant's own income.

The migrant's set of comparison groups consists of fellow migrants and natives. In order to represent the social proximity of a migrant to these two groups, we incorporate weights in the migrant's utility function. Regarding proximity to the natives as a comparison group, in a manner akin to Akerlof's (1997), we assume that the more effort a migrant exerts to assimilate into the mainstream culture, the closer he is in social space to the natives, and the more he compares himself with them. By fine-tuning the extent of his assimilation, the migrant can manage his social distance from the natives: a limited assimilation results in a weight attached to the comparison with the natives that is less than one; a maximum assimilation (which means that the distance in social space between the migrant and the natives is zero) results in a weight attached to comparison with the natives that is equal to one. Regarding proximity to fellow migrants, the weight of fellow migrants as a comparison group in the relative deprivation component of the utility function of a migrant is the complement of the weight attached to the natives as a comparison group, such that the two weights add up to one.

We denote by $p(x_i)$ the weight of being affiliated with the natives and, thus, of a migrant comparing himself with them, where $0 \leq p(x_i) \leq 1$. Formally, we define $p(\cdot)$ as a twice continuously differentiable function, such that

$$\begin{aligned} &0 < p(x_i) < 1 \text{ for } 0 < x_i < 1, \\ &p(0) = 0, \\ &p(1) = 1, \\ &\lim_{x_i \to 0} p'(x_i) < \infty, \\ &p'(x_i) > 0 \text{ for } 0 < x_i < 1, \\ &p''(x) \geq 0, \end{aligned} \tag{5}$$

where the last but one condition in (5) means that the greater the assimilation effort, the higher the weight attached to the natives as a comparison group. In expressing p as a function of x_i we make two important assumptions. First, that the level of social proximity to the group of natives is determined endogenously (namely by the choice of the assimilation effort x_i). And second, that the exertion of effort to assimilate and the level of social proximity to the natives are linked. Hence, and for example, it is not possible to choose to exert a high level of assimilation effort while at the same time exclude the natives as a comparison group.

In addition to (5), we assume that the stronger the level of identification with the natives (the closer the social proximity to the natives), the weaker the identification with fellow migrants (the greater the distance in social space from fellow migrants). This assumption is expressed by a weight $1 - p(x_i)$ that migrant i accords to comparing himself with his fellow migrants.

For migrant i, the average income of his fellow migrants is denoted by $\bar{Y}(\mathbf{x}_{-i})$, namely $\bar{Y}(\mathbf{x}_{-i}) = \frac{1}{n-1} \sum_{j \in \{1,\ldots,n\} \setminus \{i\}} Y(x_j)$. The average income of the natives is denoted by $\bar{Z}$, and is assumed not to be lower than the highest earnings achievable by a migrant, namely $Y(1) \leq \bar{Z}$. We can now express migrant's i relative deprivation as

$$RD(x_i, \mathbf{x}_{-i}) = p(x_i) F\left[\bar{Z} - Y(x_i)\right] + \left(1 - p(x_i)\right) F\left[\bar{Y}(\mathbf{x}_{-i}) - Y(x_i)\right]. \tag{6}$$

We assume that as a function of x_i, $RD(x_i, \mathbf{x}_{-i})$ remains convex. For this to hold, it suffices to assume that, for a given $Y(\cdot)$, the functions $p(\cdot)$ and $F(\cdot)$ observe specific conditions related to their convexity.[6]

[5] The convexity property follows from $\frac{d^2 F[A - Y(x_i)]}{dx_i^2} = F''[A - Y(x_i)][Y'(x_i)]^2 - F'[A - Y(x_i)]Y''(x_i) \geq 0$; when $A > Y(x_i)$, this inequality is strict.

[6] The sufficient conditions for the convexity of $RD(x_i, \mathbf{x}_{-i})$ are that $F''(x) \geq 0$, $F'''(x) \geq 0$, $p''(x) \geq 0$, and $\left[\log p'(x)\right]' > 2Y'(x)\left[\log F[a - Y(x)]\right]'$ for any $a \leq \bar{Z}$. The proof of this claim is available from the authors on request. These conditions are observed for a wide range of functions $F(\cdot)$, $p(\cdot)$, and $Y(\cdot)$.

Inserting (6) into (1), we express the utility of migrant i as

$$u_i(x_i,\mathbf{x}_{-i}) = Y(x_i) - p(x_i)F\left[\bar{Z} - Y(x_i)\right] - \left(1 - p(x_i)\right)F\left[\bar{Y}(\mathbf{x}_{-i}) - Y(x_i)\right] - c_i C(x_i). \tag{7}$$

Because the function $u_i(x_i,\mathbf{x}_{-i})$ is the sum of concave functions of x_i (namely of $Y(x_i)$, $-p(x_i)F\left[\bar{Z} - Y(x_i)\right]$, $-\left(1 - p(x_i)\right)F\left[\bar{Y}(\mathbf{x}_{-i}) - Y(x_i)\right]$, and $-c_i C(x_i)$), then it is concave in x_i.

The utility formulation in (7) expresses both the standard tension between the unpleasant exertion of effort aimed at acquiring productive "tools," and the consequent pleasing acquisition of income, as well as an additional dimension of tension, namely the added relative deprivation terms (the middle terms on the right-hand side of (7)): the effort to acquire productive "tools" results in a reduction in the displeasure that arises from a relatively low income within a comparison group, yet it increases the weight that is accorded to the natives as a comparison group, which yields substantial discontent.

3. The assimilation game: A homogenous community of migrants

In this section, we consider a community that consists of migrants who are identical with respect to their assimilation capabilities: the cost of exerting effort to assimilate is the same for each of them and is $c_i = 1$ for $i \in \{1, 2, \dots, n\}$. We show that in such a case, the equilibrium of the assimilation game can be obtained from a "reduced form" payoff function which abstracts from the effect of comparisons with fellow migrants. With the third term on the right-hand-side of (7) set equal to zero and with $c_i = 1$, the utility function $u_i(x_i,\mathbf{x}_{-i})$ simplifies to

$$u_i^{id}(x_i) \equiv Y(x_i) - p(x_i)F\left[\bar{Z} - Y(x_i)\right] - C(x_i). \tag{8}$$

The function $u_i^{id}(x_i)$ is the utility of a migrant when all the migrants choose the same level of effort so that comparisons between them do not inflict relative deprivation. Drawing on a similar reasoning to the one pertaining to $u_i(x_i,\mathbf{x}_{-i})$ above, $u_i^{id}(x_i)$ is also concave. We denote by x^* the level of assimilation that maximizes $u_i^{id}(x_i)$. The corresponding first order condition is

$$u_i^{id'}(x^*) = 0,$$

namely

$$Y'(x^*) - p'(x^*)F\left[\bar{Z} - Y(x^*)\right] + p(x^*)Y'(x^*)F'\left[\bar{Z} - Y(x^*)\right] - C'(x^*) = 0. \tag{9}$$

The concavity of the $u_i^{id}(x_i)$ function in (8) ensures that x^*, the level of effort that yields the maximal utility, is unique. In addition, the limit assumptions in (2), (3), and (5) imply that this level is strictly positive, and that it is interior: $0 < x^* < 1$.

It turns out that the symmetric solution given by the n-dimensional point $\mathbf{x}^* = (x^*, \dots, x^*)$ is the unique Nash equilibrium of the game between migrants who are identical with respect to their efficiency at assimilation. This result and a result relating to the welfare of the community of migrants, are stated in the following proposition.

Proposition 1. *The assimilation equilibrium of a homogeneous community of migrants.*
The choice of the levels of assimilation effort given by the point $\mathbf{x}^* = (x^*, \dots, x^*)$ is the unique Nash equilibrium of the homogeneous community of migrants, namely of a community characterized by payoff functions $u_i(x_i,\mathbf{x}_{-i})$ as in (7) with $c_i \equiv 1$ for $i \in \{1, 2, \dots, n\}$. This equilibrium maximizes the community's welfare, defined as the sum of the utilities of the migrants, $SU(\mathbf{x}) \equiv \sum_{i=1}^{n} u_i(x_i,\mathbf{x}_{-i})$ for $\mathbf{x} \in [0,1]^n$.

Proof. The proof is in the Appendix.

The proof of Proposition 1 reveals that the only stable equilibrium of the homogeneous community of migrants can be reached by the maximization of the "reduced form" utility function ($u_i^{id}(\cdot)$ in (8)). In addition to characterizing the equilibrium, Proposition 1 states that the equilibrium is socially optimal. Therefore, and as elaborated next, in the event of disruption, the migrant community will have an incentive to "defend" this optimal choice of the common level of assimilation.

4. The assimilation game: The appearance of a mutant migrant

In Section 3, all the n migrants were characterized by identical assimilation capabilities (their cost of exerting assimilation effort was assumed to be the same), and they all chose the same level of effort (x^*), which resulted in each of them having the same income. In such a uniform community of migrants, no relative deprivation was caused from comparisons within the community, and the migrants experienced relative deprivation only from comparisons with the richer natives.

Suppose now that a mutant migrant appears among the community of the n migrants. As already explained, this migrant is characterized by a greater ability to assimilate. We express the assimilation advantage of a mutant migrant by a lower cost of exerting assimilation effort. For example, the lower cost can derive from greater learning abilities (of the language, culture, laws, the way of conducting business, or specific skills needed and valued in the labor market of the host city).

We denote the index of the mutant migrant by $m \in \{1, \dots, n\}$. We assume that the reduced cost of assimilation of the mutant migrant, c_m, is a random variable distributed on the interval $(\underline{c}, 1)$, where $\underline{c} \in [0, 1)$, with a probability density function $g(c_m)$ such that $g(c_m) > 0$ for $c_m \in (\underline{c}, 1)$, and with a cumulative distribution function $G(c_m)$. Thus, $\underline{c}$ is the boundary to the greater ability of a mutant migrant to assimilate in comparison to the "normal" migrants.

If a migrant who can assimilate at a lower cost than the other migrants deviates from the common assimilation level x^*, he will earn more than his fellow migrants. In turn, the other migrants, experiencing relative deprivation from comparisons with him, can be expected to consider adjusting their assimilation behavior. We denote by N_{-m} the set of the non-mutant migrants (all of whom are characterized by $c_i = 1$), that is, $N_{-m} = \{1, \dots, n\} \backslash \{m\}$. As a benchmark, we formulate a proposition that describes the new assimilation equilibrium of the community of migrants reached following the appearance of a mutant community member who is not constrained by any community reaction.

Proposition 2. *The assimilation equilibrium when one of the migrants is a mutant, and when his choice of assimilation behavior is not constrained by a response from the "normal" migrants.*
When a community of migrants does not interfere with the choice of assimilation effort of a mutant migrant, then the equilibrium level of assimilation will shift from $\mathbf{x}^* = (x^*, \dots, x^*)$ (the assimilation effort exerted by every member of a homogeneous community of migrants) to a point $\tilde{\mathbf{x}}^* = (\tilde{x}_1^*, \dots, \tilde{x}_n^*)$ with $\tilde{x}_i^* \equiv \tilde{x}^*$ for $i \in N_{-m}$, and $\tilde{x}_m^*$ such that:

(i) $\tilde{x}_m^* > \tilde{x}^* > x^*$

and

(ii) $\tilde{x}_m^*$ is inversely related to the parameter c_m.

Proof. The proof is in the Appendix.

Proposition 2 states that an unhindered mutant migrant will assimilate more than "normal" migrants; mutation breeds deviation. Moreover, the greater the mutant migrant's advantage in assimilation ability (the bigger the reduction in the cost of exerting effort to assimilate), the larger the deviation. Importantly, the non-mutant migrants will also exert a greater effort than they would have exerted if a mutant had not appeared. Consequently, their utility will be lowered. This is so because (recalling (8))

$$u_i(\tilde{x}^*,\tilde{\mathbf{x}}^*_{-i}) = Y(\tilde{x}^*) - p(\tilde{x}^*)F\left[\bar{Z} - Y(\tilde{x}^*)\right] - \left(1-p(\tilde{x}^*)\right)F\left[\bar{Y}(\tilde{\mathbf{x}}^*_{-i}) - Y(\tilde{x}^*)\right] - C(\tilde{x}^*)$$
$$\leq Y(\tilde{x}^*) - p(\tilde{x}^*)F\left[\bar{Z} - Y(\tilde{x}^*)\right] - C(\tilde{x}^*) = u_i^{id}(\tilde{x}^*) < u_i^{id}(x^*)$$

for any $i \in N_{-m}$, where the last inequality in the expression above follows from the fact that x^* maximizes the simplified utility function $u_i^{id}(\cdot)$ (which, to recall, applies when there are no differences in incomes between the migrants).

Because deviation by a mutant migrant lowers the utility of his fellow migrants, they have an incentive to discourage him from increasing his level of assimilation beyond theirs. We next study the manner in which they act on this incentive.

5. The assimilation game: A community response to the appearance of a mutant migrant

In order to track the community's reaction to the appearance of the mutant migrant, we proceed in two steps. In Subsection 5.1 we outline the type of sanction that can be applied by the community of "normal" migrants to discourage the mutant migrant from deviating. To find out when sanctioning by the community is likely to be efficient to the community, we present and analyze in Subsection 5.2 a correspondingly modified game played between the migrants when a deviant migrant can be sanctioned.

5.1. Sanctioning a mutant migrant

We now assume that a community of migrants can impose a sanction: if a migrant deviates from x^*, the common level of assimilation that maximizes the utilities of "normal" migrants, they will shun him. The idea that a community can apply social pressure as a means of persuasion is not original to this paper. "Rabbeinu Tam sanctions," which date back to medieval times, amount to a community distancing itself from deviants. In the 12th century context, these deviants were men who refused to give their wives a bill of divorce, known as a "get," to make a separation official. The sanctions consisted of shunning and ostracizing, in particular refusal to trade or pray with a man who refuses to give his wife a "get," and not "giving any honors" to him.

Suppose, then, that a mutant migrant who strays from the community's assimilation norm, x^*, by exerting a higher level of effort x_m such that $x_m > x^*$, is penalized (shunned) by the community. As a result, the mutant will find himself closer in social space to the natives than would be the case had the community not reacted. Specifically, the weight that the mutant migrant will end up attaching to the natives as a comparison group will be $p(x_m) + \beta\left(1 - p(x_m)\right)$, where $\beta \in (0,1)$ is a parameter. In other words, we assume that this weight will be increased by a fraction β of the weight that hitherto the migrant has attached to his fellow migrants as a comparison group. The parameter β measures the severity of the sanction: the "revenge" of fellow migrants shunning the mutant is affected by a shift in social space onto the natives of a weight that the migrant has assigned to his fellow migrants.

In comparison with a situation in which the mutant does not deviate from the community's equilibrium level of assimilation, with deviation the weight attached to the natives increases twofold: first, from $p(x^*)$ (which is the weight that the community attaches to the natives when the equilibrium level of assimilation is x^*, as given by (9)) to $p(x_m)$ (which is the weight that is associated with the mutant migrant's heightened level of effort x_m); and, second, as a result of the community's distancing sanction, from $p(x_m)$ to $p(x_m) + \beta\left(1 - p(x_m)\right)$. Naturally, if the mutant migrant refrains from seeking enhanced assimilation, then he is not subject to a sanction. In short, the mutant migrant's utility is

$$\hat{u}_m(x_m,\mathbf{x}_{-m}) = \begin{cases} Y(x_m) - \left[p(x_m) + \beta\left(1-p(x_m)\right)\right]F\left[\bar{Z} - Y(x_m)\right] \\ \quad - \left[1 - p(x_m) - \beta\left(1-p(x_m)\right)\right] \\ \quad \cdot F\left[\bar{Y}(\mathbf{x}_{-m}) - Y(x_m)\right] - c_m C(x_m) & \text{for } x_m > x^*, \\ Y(x_m) - p(x_m)F\left[\bar{Z} - Y(x_m)\right] \\ \quad - \left(1 - p(x_m)\right)F\left[\bar{Y}(\mathbf{x}_{-m}) - Y(x_m)\right] - c_m C(x_m) & \text{for } x_m \leq x^*, \end{cases} \tag{10}$$

where $\mathbf{x}_{-m} = (x_i)$ for $i \in N_{-m}$ is the assimilation point chosen by the "normal" migrants. We note that the added factor $\beta\left(1 - p(x_m)\right)$ does not compromise the concavity of $\hat{u}_m(x_m,\mathbf{x}_{-m})$ for $x_m > x^*$.[7]

The severity of the community's sanction, reflected in the magnitude of β, depends on the cohesion of the community, namely on the ability of the members of the community to enact, coordinate, and apply an injunction that amounts to a wall without cracks or gaps.[8] In a very cohesive community, even the closest family of the deviant will shun him, so he has no choice but to get close to the natives; in a less cohesive community, only distant friends of the deviant will make him feel like a stranger, so his comparison perspective does not change that much, and the penalty that he is subjected to is not all that formidable.

It is worth mentioning that the characterization of the penalty above has two additional consequences. First, because by construction the weight attached to the natives cannot exceed one, the sanction will be of limited severity if the social proximity to the natives of the deviant migrant is already quite high (namely if $p(x_m)$ is close to 1). Second, the possibilities for penalizing a deviant are reduced if the community's initial degree of assimilation is high (namely if $p(x^*)$ itself is close to 1). In other words, when it comes to preventing a deviation, a community that to begin with chooses a close level of proximity to the natives has its hands largely tied.

5.2. Equilibrium of the community-mutant migrant assimilation game

We revise the formulation of the game in order to include the possibility of sanctioning the mutant migrant in the strategy space, and in order to characterize the manner in which the community applies the sanction. The required modification is lessened because Proposition 2 ensures that when confronted with the mutation, all the non-mutant migrants choose the same level of effort. Consequently, all the non-mutant migrants have the same preference regarding sanctioning the mutant. We thus analyze the game between two players: "the community" (of non-mutant migrants), and the mutant migrant.[9]

The modified setting consists of the following sequence of stages. At the very beginning, the mutant migrant's assimilation ability, c_m, is drawn randomly from the distribution $G(c_m)$ and becomes known to the players. Then, a two-step game is played. In step I, the community decides whether to apply the sanction (in which case the utility function of the mutant migrant is given by (10)) or, alternatively, to allow the mutant migrant to choose his level of effort unhindered (in which case his utility function is given by (7)). In step II, the community and the mutant choose their effort levels. Thus, the community adopts a strategy $s_{com} = (\gamma, x)$, where $\gamma \in \{\text{"Sanction"}, \text{"Allow"}\}$, and $x \in [0,1]$. Here, we use x instead of x_i because we can limit ourselves to symmetric choices by the non-mutant migrants. The strategy of the mutant migrant is given by $s_m = (x_m^S, x_m^A)$, where x_m^S is the assimilation effort of the mutant migrant when he is subjected to a sanction, and x_m^A is his effort level when the community does not sanction him (obviously, $\{x_m^S, x_m^A\} \in [0,1]$).

[7] To see this, we note that if the conditions listed in footnote 6 are observed, then so are the corresponding conditions where $p(x)$ is replaced by $p(x) + \beta\left(1 - p(x)\right)$.

[8] The sanctioning mechanism presented here – "shunning" of the deviant – is the only costless sanction available to the "normal" migrants in a setting based on social proximity and social comparisons.

[9] Another way of thinking about this setting is a game between a representative non-mutant migrant and the mutant.

In the following proposition we characterize the equilibria of the community-mutant game for given β and c_m.

Proposition 3. *The equilibrium of the community-mutant migrant game when a mutant migrant can be sanctioned.*

We consider the following condition:

$$u_m^{id}(x^*) \geq \sup_{\mathrm{A}} \hat{u}_m(x_m, \mathbf{x}_{-m}), \tag{11}$$

where $\mathrm{A} = \{(x_1, \ldots, x_n) : x_m > x^*, x_i = x < x_m \text{ for } i \neq m\}$. The community-mutant game presented in this section has a unique subgame perfect Nash equilibrium in pure strategies, and is of one of following two types:

(a) If condition (11) holds, then the community plays ("Sanction", x^*), and the mutant migrant plays (x^*, x_m^A), for some $x_m^A > x^*$.
(b) If condition (11) does not hold, then the community plays ("Allow", $\tilde{x}^*$), and the mutant migrant plays $(x_m^S, \tilde{x}_m^*)$, for some $x_m^S > \tilde{x}_m^*$.

Proof. The proof is in the Appendix.

Remark. In an equilibrium of type (a), the realized vector of assimilation efforts is the same as in Proposition 1, whereas in an equilibrium of type (b), the realized vector of assimilation efforts is the same as in Proposition 2.

Condition (11) depends implicitly on β and on c_m. It informs us that a mutant migrant with a given c_m who faces a sanction of magnitude β will be better off when he adheres to the common assimilation level x^* than when he deviates. As revealed by the proof of Proposition 3, if condition (11) is not met and the community applies the sanction, then the sanction will actually induce the mutant to choose a *higher* level of effort than he would choose to exert if no sanction was imposed on him. Therefore, the community plays "Sanction" only if in doing so it is able to dissuade the mutant from deviating. Administering an inefficient sanction is against the interests of the community.[10]

In the next proposition we show that the prevailing equilibrium type (be it equilibrium type (a) or equilibrium type (b), as defined in Proposition 3) depends on the severity of the sanction β and on the assimilation advantage of the mutant c_m.

Proposition 4. *Determination of the type of equilibrium of the community-mutant migrant game.*

For a given level of severity of the sanction β, there exists a critical level of advantage in assimilation ability, $c(\beta) \in (0, 1]$, such that if $c_m \geq c(\beta)$, then the equilibrium is of type (a) in Proposition 3, and if $c_m < c(\beta)$, then the equilibrium is of type (b) in Proposition 3. The critical value $c(\beta)$ is weakly declining in β.

Proof. The proof is in the Appendix.

Finally, on recalling that $\underline{c}$ is the lowest possible cost of assimilation for the mutant migrant, in the next proposition we link the cohesion of the community with its ability to dissuade a mutant migrant from deviating.

[10] A related observation is that it is important to require the equilibria to be subgame perfect. If the mutant were to play (x_m^S, x_m^A) such that $x_m^S, x_m^A > x^*$, then the best response of the community would include "Allow" in step I. A Nash equilibrium of this type exists even when condition (11) is violated, although in such a case the equilibrium is not perfect: an announcement by the mutant migrant that he will play $x_m^S > x^*$ is an empty threat.

Proposition 5. *Community cohesion and stability of the assimilation level.*

The critical level of the advantage in assimilation ability, $c(\beta)$, determines the probability that a community will succeed in preventing a mutant migrant from acting on his improved ability, $P(\beta)$, so that:

(i) if $c(\beta) \leq \underline{c}$, then the probability that the equilibrium of the game will be of type (b) is zero: $P(\beta) = 1$;
(ii) if $c(\beta) > \underline{c}$, then the probability that the equilibrium of the game will be of type (b) is greater than zero: $P(\beta) = 1 - G(c(\beta)) < 1$;
(iii) $P(\beta)$ is weakly increasing in β.

Proof. The proof is in the Appendix.

Proposition 5 maps the community's cohesion, measured by the strength of its sanction, onto effectiveness in preventing a mutant migrant from acting on his advantage in assimilation ability. Part (iii) of the proposition informs us that the more cohesive the community, the higher the probability that its sanction will be effective, that is, the outcome of the game will be equilibrium (a). Specifically (as per part (i) of Proposition 5), if the community's cohesion is such that $P(\beta) = 1$, then the community can impose a powerful enough sanction to prevent a mutant with any possible assimilation advantage from acting on his edge. We refer to such a community as a tightly knit community, meaning that this is a community that can maintain the stability of the socially optimal assimilation equilibrium (a), namely, keep the assimilation level of all members at x^*.

However, as part (ii) of Proposition 5 informs us, if the community's cohesion is such that $P(\beta) < 1$, then with probability $P(\beta)$ a mutant with a powerful enough edge in assimilation ability will be able to ignore the community's sanction. In such a case, the assimilation equilibrium will be of type (b): the community refrains from sanctioning, and assimilation proceeds as per Proposition 2, diverging from the social optimum. We describe a community which is vulnerable to the appearance of a mutant as a loose-knit community.

6. Conclusions

Our starting assumption has been that assimilation increases migrants' social proximity to the natives; closer proximity implies exposure to relative deprivation caused by more intensive comparison with the natives whose incomes are higher than those of the migrants; and distress at the prospect of relative deprivation acts as a check on the inclination to assimilate, resulting in a low equilibrium level of assimilation by the community of migrants. We then introduced a mutation (an exogenously derived enhanced ability to assimilate in one of the migrants) and cohesion of the community (the ability of the members of the community to apply a sanction in order to safeguard the equilibrium).

By tracking the stability of the assimilation equilibrium when a mutation occurs, we found that the community's level of assimilation is inversely related to its cohesion, in the sense that in a more tightly knit community a mutation has to be stronger in order to be realized. In other words, for a given mutation, a more tightly knit community is less likely to see its assimilation equilibrium disrupted than a less tightly knit community.

The mechanism leading to these occurrences is that the community of migrants can penalize a mutant migrant for exceeding the communal level of effort to assimilate. The application of a sanction by the community is similar to "Rabbeinu Tam sanctions" that date back to medieval times and amount to a community distancing itself from deviants.

An interesting issue that this paper does not tackle is how to quantify the cohesiveness of a community. One approach could be to exploit a spatial dimension of propinquity. When migrants reside in close proximity to each other, a variety of interactions, social and other, are more likely than when migrants are thinly dispersed. Similarly, if the closeness of a community is the result of geographical clustering, then deviations from a community norm are more visible than if the migrants do not live in close concentration. These considerations introduce a spatial dimension to the concepts of social cohesion and communal stress brought about by mutation.

With regard to the efficacy of policies aimed at encouraging the assimilation of migrants, we can infer from our analysis that for an assimilation-boosting policy to be effective, the policy should be either to assist *all* migrants, albeit modestly, or to focus more intensively on a *small number of* migrants so as to enable the latter to overcome the community's sanction, and consequently, by instilling relative deprivation "from the inside," prompt assimilation by the entire group of migrants, even if the group is of the tightly knit type.[11] A policy that falls in between, that is, modest effort directed at a large but limited subset of migrants, is unlikely to be successful because it will trigger a mechanism of community sanction.

Policy cannot be formulated, and society's resources should not be spent, without an understanding of the rational choices that migrants make in their host country, and of what governs those choices. We have sought to contribute to this understanding by systematically identifying paths in social space that lead migrants to exhibit distinct patterns of assimilation.

Appendix. Proofs of Propositions 1 through 5

Proof of Proposition 1

To begin with, we prove that the n-dimensional point $\mathbf{x}^* = (x^*, \dots, x^*)$, where x^* is the solution of (9), constitutes a Nash equilibrium among the homogeneous community of migrants characterized by payoff functions $u_i(x_i, \mathbf{x}_{-i})$ with $c_i = 1$ for $i \in \{1, 2, \dots, n\}$. To this end, we check whether it is beneficial for any migrant i to unilaterally increase / decrease his effort level x_i above / below x^*. Let $\mathbf{x}^*_{-i}$ be a $(n-1)$-dimensional point such that $\mathbf{x}^*_{-i} = (x^*, \dots, x^*)$.

Suppose that $x_i > x^*$. Then $Y(x_i) > \bar{Y}(\mathbf{x}^*_{-i})$, thus, as follows from (4), $F\left[\bar{Y}(\mathbf{x}^*_{-i}) - Y(x_i)\right] = 0$, and then $u_i(x_i, \mathbf{x}^*_{-i}) = u_i^{id}(x_i)$, which has a maximum when $x_i = x^*$ for $i \in \{1, 2, \dots, n\}$. Because

$$\begin{aligned}\frac{\partial u_i(x_i, \mathbf{x}^*_{-i})}{\partial x_i} &= Y'(x_i) - p'(x_i)F\left[\bar{Z} - Y(x_i)\right] + p(x_i)Y'(x_i)F'\left[\bar{Z} - Y(x_i)\right] \\ &\quad - C'(x_i) \\ &= u_i^{id\prime}(x_i) < 0,\end{aligned} \tag{A1}$$

it is not optimal for a migrant to choose $x_i > x^*$.

Suppose alternatively that $x_i < x^*$. We define a function $v(x_i, \mathbf{x}_{-i})$ as follows:

$$v(x_i, \mathbf{x}_{-i}) \equiv -\left(1 - p(x_i)\right) F\left[\bar{Y}(\mathbf{x}_{-i}) - Y(x_i)\right], \tag{A2}$$

hence

$$u_i(x_i, \mathbf{x}_{-i}) = u_i^{id}(x_i) + v(x_i, \mathbf{x}_{-i}).$$

Then,

$$\frac{\partial u_i(x_i, \mathbf{x}^*_{-i})}{\partial x_i} = u_i^{id\prime}(x_i) + \frac{\partial v(x_i, \mathbf{x}^*_{-i})}{\partial x_i}. \tag{A3}$$

From the concavity of $u_i^{id}(\cdot)$ it follows that $u_i^{id\prime}(x_i) > 0$ for any $x_i < x^*$. On the other hand, using assumptions (2), (4), and (5), it can be verified that $\frac{\partial v(x_i, \mathbf{x}^*_{-i})}{\partial x_i} > 0$.[12] By implication, we obtain that $\frac{\partial u_i(x_i, \mathbf{x}^*_{-i})}{\partial x_i} > 0$, and, thus, x^* is the unique maximum of $u_i(x_i, \mathbf{x}^*_{-i})$ with respect to x_i.

Next, we inquire whether there exists a symmetric Nash equilibrium other than $\mathbf{x}^*$. There are two cases to consider.

First, let $\mathbf{y}^*$ be a n-dimensional point such that $\mathbf{y}^* = (y^*, \dots, y^*)$ where $y^* < x^*$. We check whether it pays off for a migrant i to shift his assimilation level away from y^*. For any $x_i \geq y^*$, $F\left[\bar{Y}(\mathbf{y}^*_{-i}) - Y(x_i)\right] = 0$, thus

$$\frac{\partial u_i(x_i, \mathbf{y}^*_{-i})}{\partial x_i} = u_i^{id\prime}(x_i) > 0$$

for $x_i \in [y^*, x^*)$ and, therefore, it pays off for migrant i to increase his assimilation level above y^*. Consequently, the vector $\mathbf{y}^*$ cannot constitute a Nash equilibrium.

Second, let $\mathbf{y}^* = (y^*, \dots, y^*)$ be such that $y^* > x^*$. Again, we inquire whether migrant i has an incentive to unilaterally decrease his x_i below y^*. Reapplying (A3) we find that

$$\frac{\partial u_i(y^*, \mathbf{y}^*_{-i})}{\partial x_i} = u_i^{id\prime}(y^*) + \frac{\partial v(y^*, \mathbf{y}^*_{-i})}{\partial x_i} < 0, \tag{A4}$$

because from the concavity of $u_i^{id}(\cdot)$ it follows that $u_i^{id\prime}(y^*) < 0$, and it can be verified that $\frac{\partial v(y^*, \mathbf{y}^*_{-i})}{\partial x_i} = 0$. Consequently, it pays off for migrant i to reduce his assimilation level. By implication, $\mathbf{y}^*$ is not a Nash equilibrium.

Moreover, we can show that the game has no asymmetric equilibria. Indeed, for a point $\mathbf{x} = (x_1, \dots, x_n)$, suppose that there exists $i \in \{1, \dots, n\}$ such that $x_i > x^*$, and let h denote the migrant whose level of effort is the highest, namely $x_h = \max\{x_i\}$. Then, migrant h will have an incentive to reduce his effort level because of a reasoning similar to the argument pertaining to (A1). On the other hand, if for some individual i it holds that $x_i < x^*$, then this individual will gain by marginally increasing his effort level because of a reasoning similar to the one following (A3). Therefore, $\mathbf{x}^*$ is the unique Nash equilibrium of this game for a homogeneous community of migrants.

Finally, we prove that $\mathbf{x}^*$ constitutes a social optimum, meaning that it maximizes the function $SU(\mathbf{x}) = \sum_{i=1}^{n} u_i(x_i, \mathbf{x}_{-i})$ for $\mathbf{x} \in [0,1]^n$. The proof is by contradiction. Suppose that a point $\mathbf{y} = (y_1, \dots, y_n)$ ensures a higher aggregate utility than $\mathbf{x}^*$. We consider two possibilities.

First, suppose that $\max\{y_1, \dots, y_n\} > x^*$. Then, on the basis of an argument similar to the one leading to (A1), it is easy to see that we can increase the utility of each of the individuals from the set $I = \left\{i \in \{1, \dots, n\} : y_i = \max\{y_1, \dots, y_n\}\right\}$ by a simultaneous marginal reduction of their assimilation level, while the levels of utility of other individuals increase too (because of a decrease in their relative deprivation). Therefore, such point $\mathbf{y}$ cannot be socially optimal.

Second, suppose that $y_i \leq x^*$ for all i and that $\min\{y_1, \dots, y_n\} < x^*$. Then, on reiterating the argument leading to (A3), it can be demonstrated that we can increase the utility of each of the individuals from the set $J = \left\{i \in \{1, \dots, n\} : y_i = \min\{y_1, \dots, y_n\}\right\}$ by a simultaneous marginal increase of their assimilation level without causing a decrease in the utility of any other individual. Hence, again, point $\mathbf{y}$ cannot be socially optimal.

This reasoning leads us to conclude that $\mathbf{x}^* = (x^*, \dots, x^*)$ is the unique social optimum of the homogeneous community of migrants, as well as the unique Nash equilibrium of the game. Q.E.D.

Proof of Proposition 2

To begin with, we show that for any vector of the assimilation effort of the non-mutant migrants $\mathbf{x}_{-m}$, the best response of the mutant migrant, $x_m(\mathbf{x}_{-m})$, is strictly bigger than x^*. The first order condition for the maximization of $u_m(x_m, \mathbf{x}_{-m})$ with respect to x_m informs us that the mutant's utility is maximized if and only if

[11] In the notation of our model, the second option of the policy will be equivalent to "engineering" a change in the distribution of the parameter c_m such that $\underline{c}$ is lowered to a level below $c(\beta)$.

[12] Explicitly, $\frac{\partial v(x_i, \mathbf{x}^*_{-i})}{\partial x_i} = p'(x_i)F\left[\bar{Y}(\mathbf{x}^*_{-i}) - Y(x_i)\right] + \left(1 - p(x_i)\right)Y'(x_i)F'\left[\bar{Y}(\mathbf{x}^*_{-i}) - Y(x_i)\right]$.

$$\frac{\partial u_m(x_m,\mathbf{x}_{-m})}{\partial x_m} = Y'(x_m) - p'(x_m)F\left[\bar{Z} - Y(x_m)\right] + p(x_m)Y'(x_m)F'\left[\bar{Z} - Y(x_m)\right] + p'(x_m)F\left[\bar{Y}(\mathbf{x}_{-m}) - Y(x_m)\right] + \left(1 - p(x_m)\right)Y'(x_m)F'\left[\bar{Y}(\mathbf{x}_{-m}) - Y(x_m)\right] - c_m C'(x_m) = 0. \tag{A5}$$

On the other hand, by assumptions (2) through (5) and because $u_i^{id'}(x^*) = 0$, for any $c_m < 1$ it holds that

$$\begin{aligned}\frac{\partial u_m(x^*,\mathbf{x}_{-m})}{\partial x_m} &= Y'(x^*) - p'(x^*)F\left[\bar{Z} - Y(x^*)\right] + p(x^*)Y'(x^*)F'\left[\bar{Z} - Y(x^*)\right] + p'(x^*)F\left[\bar{Y}(\mathbf{x}_{-m}) - Y(x^*)\right] \\ &\quad + \left(1 - p(x^*)\right)Y'(x^*)F'\left[\bar{Y}(\mathbf{x}_{-m}) - Y(x^*)\right] - c_m C'(x^*) \\ &= u_i^{id'}(x^*) + p'(x^*)F\left[\bar{Y}(\mathbf{x}_{-m}) - Y(x^*)\right] + \left(1 - p(x^*)\right)Y'(x^*)F'\left[\bar{Y}(\mathbf{x}_{-m}) - Y(x^*)\right] + (1 - c_m)C'(x^*) > 0.\end{aligned} \tag{A6}$$

Hence, by the concavity of $u_m(x_m, \mathbf{x}_{-m})$ with respect to x_m, we obtain that

$$x_m(\mathbf{x}_{-m}) > x^*, \tag{A7}$$

namely, indeed the mutant does deviate by choosing a level of assimilation that is higher than x^* for any $\mathbf{x}_{-m}$.

Next, we show that in any equilibrium of the game, the mutant's assimilation effort is the biggest in the community of migrants. Suppose, on the contrary, that $\tilde{\mathbf{y}}^* = (y_1^*, \dots, y_n^*)$ is an equilibrium and that $\tilde{y}_j^* = \max\{\tilde{y}_1^*, \dots, \tilde{y}_n^*\}$ for some $j \neq m$. Then, a non-mutant migrant j who exerts the assimilation effort $\tilde{y}_j^*$ does not sense relative deprivation from comparison with other migrants, and $u_j(\tilde{y}_j^*, \tilde{\mathbf{y}}_{-j}^*) = u_j^{id}(\tilde{y}_j^*)$. But $\tilde{\mathbf{y}}^*$ is an equilibrium, so $\tilde{y}_j^*$ has to be individual's j optimal choice, that is, $u_i^{id'}(\tilde{y}_j^*) = 0$, implying that $\tilde{y}_j^* = x^*$ which, given (A7) and the assumption that $y_j^* > \tilde{y}_m^*$, leads to contradiction.

Because the assimilation effort of the mutant migrant is the highest in the migrant community, other migrants do not inflict relative deprivation on him and, thus, his utility can be expressed by the following function (recalling (8)):

$$u_m^{id}(x_m) = Y(x_m) - p(x_m)F\left[\bar{Z} - Y(x_m)\right] - c_m C(x_m). \tag{A8}$$

As argued in Section 3, the function $u_m^{id}(\cdot)$ has a unique maximum. We conclude that in any equilibrium of the game, $\tilde{\mathbf{x}}^* = (\tilde{x}_1^*, \dots, \tilde{x}_n^*)$, the mutant migrant chooses the level of assimilation effort $\tilde{x}_m^*$ such that $u_m^{id'}(\tilde{x}_m^*) = 0$.[13]

To find the remaining elements of $\tilde{\mathbf{x}}^*$, we proceed as in Section 3, and we first look at the symmetric best responses to $\tilde{x}_m^*$. If all the non-mutant migrants choose the same level of effort $x_i = x$ for $i \in N_{-m}$, then the average income of the migrants who constitute the comparison group of individual i is $\bar{Y}(\tilde{\mathbf{x}}_{-i}) = \frac{n-2}{n-1}Y(x) + \frac{1}{n-1}Y(\tilde{x}_m^*)$, thus $\bar{Y}(\tilde{\mathbf{x}}_{-i}) - Y(x) = \frac{1}{n-1}Y(\tilde{x}_m^*) - \frac{1}{n-1}Y(x)$, and the utility of each of the non-mutant migrants can be written as

$$\tilde{u}^{id}(x) \equiv Y(x) - p(x)F\left[\bar{Z} - Y(x)\right] - \left(1 - p(x)\right)F\left[\frac{1}{n-1}Y(\tilde{x}_m^*) - \frac{1}{n-1}Y(x)\right] - C(x). \tag{A9}$$

We note that $\tilde{u}^{id}(\cdot)$ is a function of one argument only, because we regard $\tilde{x}_m^*$ as given, and because we consider only symmetric choices of assimilation effort among non-mutants. Furthermore, $\tilde{u}^{id}(\cdot)$ is concave, and the assumptions about $F(\cdot)$, $p(\cdot)$, and $C(\cdot)$ ensure that $\tilde{u}^{id}(\cdot)$ has a unique maximum, which we denote by $\tilde{x}^*$. We can now apply the same logic as in the proof of Proposition 1 (equations (A1) through (A4) and the discussion that follows) to show that the vector of the levels of assimilation effort

$$\tilde{\mathbf{x}}^* = (\tilde{x}^*, \dots, \tilde{x}_m^*, \dots, \tilde{x}^*)$$

such that the mutant chooses $\tilde{x}_m^*$ and all the non-mutants choose $\tilde{x}^*$, is a Nash equilibrium, and no other Nash equilibria exists.

To show that $\tilde{x}^* > x^*$, namely that the non-mutant migrants follow the mutant in deviating from the equilibrium x^* of the homogenous case, we note that

$$\tilde{u}^{id}(x) = u_i^{id}(x) - \left(1 - p(x)\right)F\left[\frac{1}{n-1}Y(\tilde{x}_m^*) - \frac{1}{n-1}Y(x)\right], \tag{A10}$$

hence

$$\tilde{u}^{id'}(x) = u_i^{id'}(x) + p'(x)F\left[\frac{1}{n-1}Y(\tilde{x}_m^*) - \frac{1}{n-1}Y(x)\right] + \frac{1}{n-1}\left(1 - p(x)\right)Y'(x)F'\left[\frac{1}{n-1}Y(\tilde{x}_m^*) - \frac{1}{n-1}Y(x)\right]. \tag{A11}$$

Recalling that $u_i^{id'}(x^*) = 0$, we obtain that

$$\tilde{u}^{id'}(x^*) = p'(x^*)F\left[\frac{1}{n-1}Y(\tilde{x}_m^*) - \frac{1}{n-1}Y(x^*)\right] + \frac{1}{n-1}\left(1 - p(x^*)\right)Y'(x^*)F'\left[\frac{1}{n-1}Y(\tilde{x}_m^*) - \frac{1}{n-1}Y(x^*)\right] > 0, \tag{A12}$$

where the inequality follows from assumptions (2) through (5) and from the fact that $\tilde{x}_m^* > x^*$. This completes the proof of part (i) of Proposition 2.

To prove part (ii) of the proposition, we apply the implicit function theorem to the equality

$$u_m^{id'}(\tilde{x}_m^*) = 0 \tag{A13}$$

and obtain

$$\frac{d\tilde{x}_m^*}{dc_m} = -\frac{\frac{du_m^{id'}(\tilde{x}_m^*)}{dc_m}}{u_m^{id''}(\tilde{x}_m^*)} = \frac{C'(\tilde{x}_m^*)}{u_m^{id''}(\tilde{x}_m^*)} < 0, \tag{A14}$$

where the inequality in (A14) holds true because by (3) we have that $C'(\cdot) > 0$, and because $u_m^{id}(\cdot)$ is a concave function. This completes the proof of part (ii) of Proposition 2. Q.E.D.

Proof of Proposition 3

While quite obviously there are an infinite number of strategy profiles, we can group them in eight categories, depending on whether the community decides to play "Sanction" or "Allow" in step I, and depending on whether the mutant migrant chooses a level of effort higher than x^* (he deviates) or not in any of the circumstances in which he finds himself in step II (namely depending on whether the community's decision in step I was "Sanction" or "Allow"). This grouping is represented in Table 1.

We use Table 1 to identify all the subgame perfect Nash equilibria of the game. In fact, before proving that the strategy profiles specified in Proposition 3 are the only equilibrium strategy profiles, we pinpoint which categories of profiles cannot represent any equilibria.

To begin with, we note that by Proposition 2 the strategy profile types (i), (ii), (v), and (vi) cannot constitute Nash equilibria: if the community refrains from sanctioning the mutant migrant in step I, he will deviate in step II, namely his best response always includes $x_m^A > x^*$.

We next show that we can eliminate type (vii) as a candidate of equilibrium.

[13] In general, it could occur that $u_m^{id'}(\tilde{x}_m^*) < 0$, but it could be possible only if $\tilde{x}_m^* = \tilde{x}_j^*$ for some $j \neq m$, an option that has been ruled out in the preceding part of this proof.

Table 1
Types of strategy profiles of the community-mutant game.

	$x_m^S \le x^*$, $x_m^A \le x^*$	$x_m^S > x^*$, $x_m^A \le x^*$	$x_m^S \le x^*$, $x_m^A > x^*$	$x_m^S > x^*$, $x_m^A > x^*$
"Sanction"	~~*(i)*~~	~~*(ii)*~~	*(iii)*	~~*(iv)*~~
"Allow"	~~*(v)*~~	~~*(vi)*~~	~~*(vii)*~~	*(viii)*

Note: Crossed numbers indicate categories that contain no subgame perfect Nash equilibria (consult the proof).

First, we note that for a given level of effort by mutant migrant, x_m, the utility of each community member who chooses his best response to x_m is decreasing in x_m. Let x_m be the mutant migrant's effort in step II, and let $x(x_m)$ be the best response of the community in step II (noting that this best response effort level does not depend on the community's decision to sanction or not to sanction in step I). To investigate how the utility of a non-mutant is influenced by x_m, we revisit function $\tilde{u}^{id}(\cdot)$ defined in (A9) in the proof of Proposition 2. On applying the envelope theorem to $\tilde{u}^{id}\left(x(x_m)\right)$, we find that the relationship of interest is given by

$$\frac{d\tilde{u}^{id}}{dx_m} = -\frac{1}{n-1}\left(1-p(x)\right)Y'(x_m)F'\left[\frac{1}{n-1}Y(x_m) - \frac{1}{n-1}Y(x)\right] < 0, \quad \text{(A15)}$$

where the strict inequality in (A15) follows from the fact that if the community plays "Allow," then by the proof of Proposition 2, $x_m^A > x$ at the optimum. An implication of (A15) is that the community will want the mutant migrant to choose less effort.

Second, suppose that the mutant migrant plays $x_m^S \le x^*, x_m^A > x^*$, so $x_m^S < x_m^A$. In this case, the community is better off when playing "Sanction" rather than "Allow." By implication, category (vii) contains no equilibria.

Even when the mutant migrant is sanctioned, he will certainly not choose a level of effort that is lower than the level of effort chosen by the "normal" migrants. To see this, we construct a function

$$u_m^s(x_m, \mathbf{x}_{-m}) \equiv Y(x_m) - \left[p(x_m) + \beta\left(1-p(x_m)\right)\right]F\left[\bar{Z} - Y(x_m)\right] - \left[1 - p(x_m) - \beta\left(1-p(x_m)\right)\right]F\left[\bar{Y}(\mathbf{x}_{-m}) - Y(x_m)\right] - c_m C(x_m)$$

for $x_m \in [0,1]$. Namely $u_m^s(x_m, \mathbf{x}_{-m})$ is the "top" formula of $\hat{u}_m(x_m, \mathbf{x}_{-m})$ in (10), defined on an unrestricted interval of choices of assimilation effort. From a discussion akin to that which follows (A8) in the proof of Proposition 2 regarding the function $u_m^s(x_m, \mathbf{x}_{-m})$, it can be inferred that when the mutant migrant is subjected to a penalty, he will still choose a level of effort that is the highest amongst the community of migrants. Consequently, the mutant migrant will not experience relative deprivation from comparisons with other migrants, and at any equilibrium of the game his utility will be a function of only one variable – his level of effort. We conclude that when the mutant migrant is sanctioned and chooses a level of assimilation effort that is higher than x^*, his utility will be given by

$$\hat{u}_m^{id}(x_m) \equiv Y(x_m) - \left[p(x_m) + \beta\left(1-p(x_m)\right)\right]F\left[\bar{Z} - Y(x_m)\right] - c_m C(x_m). \quad \text{(A16)}$$

We also note that $\hat{u}_m^{id}(\cdot)$ is a concave function.[14]

We consider the following difference

$$\hat{u}_m^{id}(x_m) - u_m^{id}(x_m) = -\beta\left(1-p(x_m)\right)F\left[\bar{Z} - Y(x_m)\right]. \quad \text{(A17)}$$

Suppose that there is an equilibrium in category (viii), and let (x_m^{S*}, x_m^{A*}) be the equilibrium strategy of the mutant migrant. By differentiating (A17) we get

$$\hat{u}_m^{id'}(x_m^{A*}) - u_m^{id'}(x_m^{A*}) = \beta p'(x_m^{A*})F\left[\bar{Z} - Y(x_m^{A*})\right] + \beta\left(1-p(x_m^{A*})\right)Y'(x_m^{A*})F'\left[\bar{Z} - Y(x_m^{A*})\right]. \quad \text{(A18)}$$

We note that by the definition of x_m^{A*}, $u_m^{id'}(x_m^{A*}) = 0$. By implication, we obtain that

$$\hat{u}_m^{id'}(x_m^{A*}) = \beta p'(x_m^{A*})F\left[\bar{Z} - Y(x_m^{A*})\right] + \beta\left(1-p(x_m^{A*})\right)Y'(x_m^{A*})F'\left[\bar{Z} - Y(x_m^{A*})\right] > 0, \quad \text{(A19)}$$

where the inequality in (A19) is implied by assumptions (2) through (5).

Suppose now that there is an equilibrium in category (iv) and, again, let (x_m^{S*}, x_m^{A*}) be the equilibrium strategy of the mutant migrant. We have that $\hat{u}_m^{id'}(x_m^{S*}) = 0$ which, given (A19) and the concavity of the $\hat{u}_m^{id}(\cdot)$, implies that $x_m^{A*} < x_m^{S*}$. Thus, we have shown that when the mutant migrant seeks to maximize $\hat{u}_m^{id}(\cdot)$, he will choose a higher level of effort in (viii) than in (iv). In terms of inequality (A15), the community will be worse off when the mutant is sanctioned, so the community will play "Allow." Type (iv) of the strategy profile can thus be eliminated.

To complete the proof, we show that the strategy profiles specified in Proposition 3 are, in fact, subgame perfect Nash equilibria, and that no other equilibria exist.

First, suppose that $u_m^{id}(x^*) \ge \sup_{\mathrm{A}} \hat{u}_m(x_m, \mathbf{x}_{-m})$, in other words condition (11) is observed. If the community plays "Sanction" in step I, then the best response of the mutant migrant is not to deviate, that is, his best response is to choose $x_m^S = x^*$,[15] and as we know from Proposition 2, $x_m^A > x^*$ (in fact, $x_m^A = \tilde{x}_m^*$). The best response of the community to this strategy of the mutant migrant is to play "Sanction" in step I and, by the proof of Proposition 1, to play $x = x^*$ in step II. Hence, this strategy profile is a Nash equilibrium. In terms of Table 1, this equilibrium strategy profile is in category (iii). By the same arguments that were invoked in the proof of Proposition 2, there are no other equilibria in this category.

Second, suppose that $u_m^{id}(x^*) < \sup_{\mathrm{A}} \hat{u}_m(x_m, \mathbf{x}_{-m})$, namely that condition (11) is violated. Then, the mutant migrant will deviate regardless of whether he is sanctioned or not. Taking into account the fact that $x_m^A < x_m^S$ and the relationship (A15), the best strategy of the community in such a case is to play "Allow." And if the community plays "Allow," then, by Proposition 2, the only Nash equilibrium of the subgame is when the mutant plays $x_m^A = \tilde{x}_m^*$ while the non-mutants play $x = \tilde{x}^*$. The strategy of the mutant migrant in the other subgame (when he is penalized) is to exert the level of effort that maximizes $\hat{u}_m^{id}(\cdot)$ as given by (A16), and we know from the earlier part of the proof that $x_m^A < x_m^S$. This equilibrium is in category (viii) and, again, it is the only perfect Nash equilibrium in this category.

Having shown that there are no subgame perfect Nash equilibria other than the ones in categories (iii) and (viii), we have exhausted all the possibilities for equilibria of the game. Q.E.D.

Proof of Proposition 4

We use the symbol $x_m^S(\beta, c_m)$ to denote the choice made by the sanctioned mutant migrant in step II of the game when the severity of the community's sanction is β, and the mutant migrant's assimilation ability parameter is equal to c_m. Let $c(\beta)$ be defined as follows:

$$c(\beta) = \inf\left\{c : x_m^S(\beta, c) = x^*\right\},$$

where x^* is the equilibrium of the homogenous case (recalling Proposition 1).

The set $\left\{c : x_m^S(\beta, c) = x^*\right\}$ is non-empty because, as can be easily verified, for $c = 1$ and any β we have that $x_m^S(\beta, c) = x^*$. Furthermore, if $c_m > c(\beta)$ then condition (11) is met and, by Proposition 3, we get the equilibrium of type (a); on other hand and if $c_m \le c(\beta)$, then the equilibrium of type (b) prevails.

[14] In the discussion that follows equation (10) we argue that $\hat{u}_m(x_m, \mathbf{x}_{-m})$ is a concave function of x_m for any $\mathbf{x}_{-m}$, thus $\hat{u}_m^{id}(x_m) = \hat{u}_m(x_m, \mathbf{x}_{-m})$ for $x_m > \max(\mathbf{x}_{-m})$ is also concave.

[15] It can be easily verified, by reformulating (A5) through (A8), that the mutant will never choose $x_m^S < x^*$.

To show that $c(\beta)$ is weakly decreasing in β, we note that

$$c(\beta) = \inf\left\{c : x_m^S(\beta, c) = x^*\right\} = \inf\left\{c : u_m^{id}(x^*) \geq \sup_{\mathrm{A}} \hat{u}_m(x_m, \mathbf{x}_{-m})\right\},$$

where $\mathrm{A} = \left\{(x_1, \ldots, x_n) : x_m > x^*, x_i = x < x_m \text{ for } i \neq m\right\}$.

We treat the utility function of a mutant migrant who is subjected to a sanction, $\hat{u}_m(x_m, \mathbf{x}_{-m})$, as a function of β, and we adopt the notation $\hat{u}_{m,\beta}(x_m, \mathbf{x}_{-m})$. From (10) we see that for $x_m > x^*$ the function $\hat{u}_{m,\beta}(x_m, \mathbf{x}_{-m})$ is decreasing in β, which entails that $\sup_{\mathrm{A}} \hat{u}_{m,\beta_1}(x_m, \mathbf{x}_{-m}) > \sup_{\mathrm{A}} \hat{u}_{m,\beta_2}(x_m, \mathbf{x}_{-m})$ for $\beta_1 < \beta_2$, while $\hat{u}_m(x^*, \mathbf{x}_{-m})$ is constant in β. Hence,

$$\left\{c : u_{m,\beta_1}^{id}(x^*) \geq \sup_{\mathrm{A}} \hat{u}_m(x_m, \mathbf{x}_{-m})\right\} \subset \left\{c : u_{m,\beta_2}^{id}(x^*) \geq \sup_{\mathrm{A}} \hat{u}_m(x_m, \mathbf{x}_{-m})\right\},$$

implying that $c(\beta_2) \leq c(\beta_1)$. Q.E.D.

Proof of Proposition 5

Recalling the assumption regarding the probability distribution of c_m, the probability that c_m will be smaller than or equal to $c(\beta)$ is $G\left(c(\beta)\right)$. To prove part (i) of the proposition, we assume that $c(\beta) \leq \underline{c}$. Thus, $G\left(c(\beta)\right) = 0$: the probability that the assimilation advantage of the mutant migrant, c_m, will be smaller than or equal to $c(\beta)$ is zero. Put differently, expressing the probability that the community will succeed in blocking the mutant migrant from acting on his improved ability as $P(\beta) = 1 - G\left(c(\beta)\right)$, we get that for $c(\beta) \leq \underline{c}$, $P(\beta) = 1$. This concludes the proof of part (i) of the proposition.

To prove part (ii) of the proposition, we note that under the assumption that $c(\beta) > c_0$, we get that $G\left(c(\beta)\right) > 0$, and that $P(\beta) = 1 - G\left(c(\beta)\right) > 0$.

To prove part (iii) of the proposition, we recall from Proposition 2 that $c(\beta)$ is weakly declining in β, which implies that for $\beta_1 < \beta_2$ we get that $c(\beta_1) \geq c(\beta_2)$. In turn, from the monotonicity property of the cumulative distribution function $G(c)$ we get that $G\left(c(\beta_1)\right) \geq G\left(c(\beta_2)\right)$, which is equivalent to $P(\beta_1) \leq P(\beta_2)$. Q.E.D.

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