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By GERHARD SORGER, ODED STARK and YONG WANG

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Migration and dynamics: How a leakage of human capital lubricates the engine of economic growth

Gerhard Sorger [a], Oded Stark [b,*], Yong Wang [c]

[a] Department of Economics, University of Vienna, Hohenstaufengasse 9, 1010 Vienna, Austria
[b] Universities of Bonn, Klagenfurt, Vienna, and Warsaw
[c] Department of Economics and Finance, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Kowloon, Hong Kong, China

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ABSTRACT

This paper studies the growth dynamics of a developing country under migration. Assuming that human capital formation is subject to a strong enough, positive intertemporal externality, the prospect of migration will increase growth in the home country in the long run. If the external effect is less strong, there exists at least a level effect on the stock of human capital in the home country. In either case, the home country experiences a welfare gain, provided that migration is sufficiently restrictive. These results, obtained in a dynamic general equilibrium setting, extend and strengthen the results of Stark and Wang (2002) obtained in the context of a static model.

1. Introduction

In an earlier paper, Stark and Wang (2002) presented a new paradigm in the study of the effects of the migration of skilled workers. Stark and Wang (2002) argued that the prospect of migration from a developing country to a developed, technologically advanced country changes not only the set of opportunities that individuals in the developing country face, but also the structure of the incentives that they confront: higher prospective returns to human capital in the developed country induce more human capital formation in the developing country. In particular, they showed that an improvement in the incentives that govern the decision to acquire human capital can lead not only to an increase in the human capital that individuals choose to form but also, under certain conditions, to a welfare gain for all, migrants and non-migrants alike. However, the static framework employed in Stark and Wang (2002) fell short of informing us whether the human capital gain can be the harbinger of long-term economic growth. This is the subject of the present paper.

We develop an overlapping-generations growth model and investigate the associated dynamic general equilibrium. We proceed in four steps. First, we derive the long-run steady state in a small open economy with free international capital flows, but without the possibility of migration. We show that this steady state is characterized by a constant level of human capital and a constant growth rate of output which, in turn, is equal to the exogenous growth rate of the population.

* Corresponding author at: ZEF, University of Bonn, Walter-Flex-Strasse 3, D-53113 Bonn, Germany.
E-mail addresses: gerhard.sorger@univie.ac.at (G. Sorger), ostark@uni-bonn.de (O. Stark), EFYWANG@cityu.edu.hk (Y. Wang).

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Second, we study growth dynamics when the individuals in the home country face a strictly positive probability of migrating to a destination country where the returns to their human capital are higher than at home. The higher expected wage rate yielded by the prospect of migration induces the individuals to acquire more human capital. In our model, human capital formation is subject to a positive intertemporal externality beyond some threshold level. We show that the size of the spillover effect of the current human capital on future productivity plays an important role in determining the long-run growth effect of the prospect of migration. When the spillover effect is sufficiently strong, the prospect of migration results in a higher growth rate of the home country in the long run; however, a weak enough, or a complete absence of, a spillover effect will lead to only a level increase in human capital, with a reduction of the growth rate in the long run.[1]

Third, we conduct a welfare analysis, looking at the wellbeing of the individuals who stay behind in the home country. When the workforce is homogeneous, these individuals too have responded (ex ante) to the migration-conferred incentive to acquire more human capital, yet ended up (ex post) not subjecting their improved human capital to the higher pay environment abroad. Provided that migration is restricted, the long-run growth gain translates into a welfare gain. Moreover, even if there is no long-run growth gain, a welfare gain is still possible under restrictive migration policies, just as originally demonstrated in Stark and Wang (2002) in the context of a static model.

Finally, in step four we demonstrate that our findings are robust to the introduction of heterogeneity of skill levels. Specifically, we assume that individuals may differ in their ability to form human capital, and that only the highly skilled individuals, who in equilibrium acquire more human capital than the low-skill individuals, face a prospect of migration. Despite the negative effect caused by the migration of such individuals, a growth gain as well as a resulting welfare gain in the home country can still materialize.

2. Human capital formation and economic growth: the (benchmark) economy without migration

Consider a small open economy without migration. The economy consists of overlapping-generations, with each generation consisting, in turn, of homogenous individuals whose lives can be divided into two periods. The population grows at a (gross) rate $n > 0$. Each member of each generation acquires human capital in the first period of his life ("youth"). In the second period of his life ("old age") the member works, repays the loan that he took to finance the human capital investment, procreates (at the rate n), and consumes.

More specifically, in each period a young individual undertakes educational investment which is financed by borrowing from a perfect capital market, where the prevailing world (gross) interest rate is $R > 0$. An old individual works, supplying inelastically the human capital which he acquired during his youth. Let e_t be the amount of educational investment, financed by borrowing, of a young individual in period t, and let h_{t+1} be the resulting level of human capital available to the individual in the subsequent period $t + 1$. We assume that the production function of human capital is a product of two terms: the young individual's own educational investment, and the human capital level of the old (parent) generation. Because within a given generation individuals are identical,[2] the prevailing average level of human capital is the same as an old individual's level of human capital. Formally, we assume that

$$h_{t+1} = {e_t}^{\alpha} {h_t}^{\beta}, \tag{1}$$

where α and β are positive parameters satisfying $\alpha + \beta < 1$ (that is, the production function of human capital exhibits decreasing returns to scale). The incorporation of h_t in (1) emanates from the assumption that the prevailing (average) level of human capital creates an environment that facilitates human capital formation for any given level of educational investment. The prevailing level of human capital thus acts as a contemporaneous pulling up externality.[3]

The output produced in the economy in period t, Y_t, is given by

$$Y_t = A_t H_t, \tag{2}$$

where H_t is the aggregate human capital in the economy (the sum of the levels of human capital of all the old individuals), and where A_t is a productivity factor. We assume that the productivity factor is subject to a threshold externality in the spirit of Azariadis and Drazen (1990), that is,

$$A_t = f(h_{t-1}), \tag{3}$$

[1] The idea that migration might lead to higher growth through some sort of an intertemporal spillover effect on productivity has been suggested, but not explicitly studied, by Mountford (1997). It is noteworthy that in contrast to Mountford (1997), who focuses on the possibility of migration resulting in some long-run *level* effects on human capital accumulation and on output for the home country, we study in the present paper the long-run *growth* as well as *welfare* effects of migration. Stark, Helmenstein, and Prskawetz (1998) have also referred to the positive externalities that the probability of migration could confer upon the home country.

[2] The case of heterogeneous individuals is studied in Section 5.

[3] Similar production functions of human capital are quite standard in the relevant literature. See, for example, Lucas (1988), Galor and Stark (1994), and Glomm and Ravikumar (2001). The restriction $\alpha + \beta < 1$ is helpful in obtaining a steady state for the human capital dynamics.

with

$$f(h_{t-1}) = \begin{cases} A & \text{for } h_{t-1} < \overline{h} \\ A\left(h_{t-1}/\overline{h}\right)^{\rho} & \text{for } h_{t-1} \geq \overline{h}, \end{cases}$$

where A, $\overline{h}$, and ρ are positive parameters. The parameter ρ is a measure of the impact of the intertemporal externality of human capital on productivity when the level of human capital is above the threshold level of human capital $\overline{h}$. Thus, the productivity factor is a continuous function of the average level of human capital in the economy, h_{t-1}. It is constant for average human capital levels below some threshold level, $\overline{h}$, and it depends positively on h_{t-1} beyond this threshold level.[4]

Before proceeding, we summarize the parameter restrictions imposed thus far.

Assumption 1. The parameters α, β, ρ, n, A, $\overline{h}$, and R are strictly positive, and it holds that $\alpha + \beta < 1$.

Let L_t be the size of the economy's labor force (the number of the old individuals) in period t, and let N_t be the size of the continuum of the young individuals born in period t. Then, $L_{t+1} = N_t = nL_t$. Denote by w_t the returns to human capital (the wage rate per unit of human capital) in period t. From (2) it follows that $w_t = A_t$. Denoting by c_{t+1} the consumption in period $t+1$ of an individual born in period t, the individual solves the following problem:

$$\max_{e_t} c_{t+1} = w_{t+1}h_{t+1} - R \cdot e_t$$

subject to (1). The first-order condition for this maximization problem yields, as a unique solution,

$$e_t = \left(\alpha w_{t+1}/R\right)^{1/(1-\alpha)} h_t^{\beta/(1-\alpha)}, \tag{4}$$

$$c_{t+1} = (1-\alpha) w_{t+1}{}^{1/(1-\alpha)} (\alpha/R)^{\alpha/(1-\alpha)} h_t^{\beta/(1-\alpha)}, \tag{5}$$

and

$$h_{t+1} = \left(\alpha w_{t+1}/R\right)^{\alpha/(1-\alpha)} h_t^{\beta/(1-\alpha)}. \tag{6}$$

From Eq. (5) it follows that consumption is non-negative, even though we did not impose this as a constraint. This property implies that the individuals are able to survive after fully repaying their loans.

Because $w_{t+1} = A_{t+1}$, substituting (3) into (6) we obtain the transition dynamics of human capital

$$h_{t+1} = H(h_t) \equiv \begin{cases} \left(\dfrac{\alpha A}{R}\right)^{\alpha/(1-\alpha)} h_t^{\beta/(1-\alpha)} & \text{for } h_t < \overline{h}; \\ \left(\dfrac{\alpha A}{R\overline{h}^{\rho}}\right)^{\alpha/(1-\alpha)} h_t^{(\alpha\rho+\beta)/(1-\alpha)} & \text{for } h_t \geq \overline{h}. \end{cases} \tag{7}$$

We denote by $\phi(h_t) \equiv \left(\dfrac{\alpha A}{R}\right)^{\alpha/(1-\alpha)} h_t^{\beta/(1-\alpha)}$ and by $\varphi(h_t) \equiv \left(\dfrac{\alpha A}{R\overline{h}^{\rho}}\right)^{\alpha/(1-\alpha)} h_t^{(\alpha\rho+\beta)/(1-\alpha)}$, that is, the first and second segments of the human capital transition dynamics in (7), respectively. It is easy to see that the dynamics represented by $\phi(\cdot)$, which is a concave function, monotonically converges to a steady state of human capital given by

$$h^* = (\alpha A/R)^{\alpha/(1-\alpha-\beta)}. \tag{8}$$

On the other hand, the function $\varphi(\cdot)$ can display varied shapes and the corresponding dynamics may have different properties, depending on parameter configurations (in particular the size of ρ). Specifically, defining $\eta \equiv \alpha(1+\rho) + \beta$, we have that $\varphi(\cdot)$ is concave if $\eta < 1$, convex if $\eta > 1$, and linear if $\eta = 1$. Furthermore, as long as $\eta \neq 1$, $\varphi(\cdot)$ has a unique steady state at

$$h^{**} = \left(\alpha A/R\overline{h}^{\rho}\right)^{\alpha/(1-\eta)}, \tag{9}$$

which is stable under the dynamics $h_{t+1} = \varphi(h_t)$ if $\varphi(\cdot)$ is concave ($\eta < 1$), and unstable if $\varphi(\cdot)$ is convex ($\eta > 1$).

In order to flesh out our main argument, we make the following assumption.[5]

Assumption 2. It holds that $\overline{h} > (\alpha A/R)^{\alpha/(1-\alpha-\beta)}$.

This assumption means that the threshold level of human capital at which the intertemporal externality in productivity takes effect is relatively high. In such a case, the economy will be approaching the steady state of human capital given by (8), h^*, from all

[4] Here it is assumed that the total productivity at time t is affected by the average level of human capital of the preceding period $t-1$. Our analysis will go through if, instead, we were to assume that total productivity depends on the contemporaneous average level of human capital.

[5] In Appendix B we show that the qualitative results of the ensuing analysis carry over to the complementary case in which Assumption 2 does not hold.

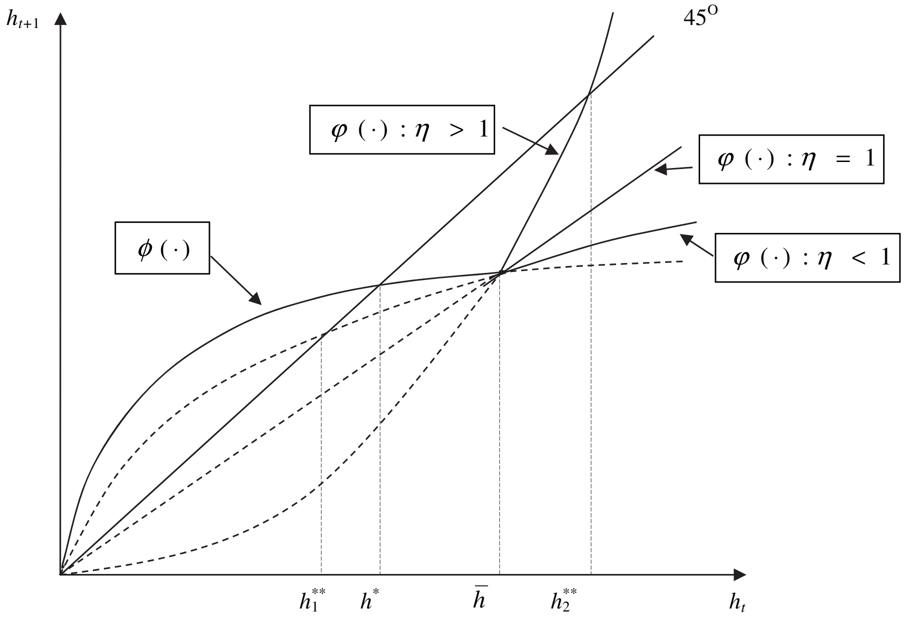

Fig. 1. The phase diagram of the human capital dynamics in the benchmark economy (when $\bar{h} > (\alpha A/R)^{\alpha/(1-\alpha-\beta)}$).

initial human capital stocks smaller than $\bar{h}$ (and also from many others). Hence, the economy is prevented from taking off to a path of persistent growth. The following lemma substantiates this intuition.

Lemma 1. The average human capital in the closed economy converges to h^*, either for any initial level of human capital h_0 if $\eta \le 1$, or for $h_0 < \left(\alpha A/R\bar{h}^{\rho}\right)^{\alpha/(1-\eta)}$ if $\eta > 1$.

Proof. See Appendix A.

The phase diagram of the human capital dynamics is illustrated in Fig. 1.

It follows from Lemma 1 that, barring the exceptional case with a rather high level of initial human capital and a large intertemporal externality,[6] the level of human capital in the closed economy will converge to $h^* < \bar{h}$, and hence, the productivity of the human capital A_t will be constant at the level of A in the long run. For sufficiently large t, the (gross) rate of growth of aggregate output in the closed economy in period t is given by

$$g_t = Y_{t+1}/Y_t = A_{t+1}L_{t+1}h_{t+1}/(A_tL_th_t) = nh_{t+1}/h_t = n(\alpha A/R)^{\alpha/(1-\alpha)}h_t^{(\alpha+\beta-1)/(1-\alpha)}$$

where the last equality in the displayed expression follows from (7) for $h_t < \bar{h}$. Because h_t converges to h^*, which is given by (8), the growth rate of the *aggregate* output converges to the steady state growth rate n, that is, $g_t \to g^* = n$ for $t \to \infty$, which is the (gross) rate of population growth. In other words, in per capita terms, the growth rate of output converges to zero in the long run.

In sum: our analysis shows that the benchmark economy without migration converges to a long-run equilibrium with the average (per member of the population) level of human capital given by (8), and with output growing at the population growth rate n. In particular, in the closed economy, the human capital formed by the individuals will be “trapped” below the threshold level $\bar{h}$ in the long run. The potential pull of the human capital externality on productivity will not come into play and consequently, as diminishing returns take over, growth is muted in the long run.

3. Human capital formation and economic growth: the economy with migration

For each and every generation, let there be a strictly positive probability, p, of migrating to a foreign country in the second period of an individual's life.[7] Thus, an individual born in period t faces a probability p of obtaining in the foreign country returns

[6] It is clear from the proof of Lemma 1 and Fig. 1 that for the closed economy to escape the “trap” of h^*, it is necessary that $\eta > 1$ and $h_0 > h_2^{**} > \bar{h}$ hold. Given our focus on the perspective of a developing economy, throughout this paper we disregard this exceptional case which requires an initial level of human capital that is conceivably far too high.

[7] As we note later on, our main results carry through even when the prospect of migration avails itself only for a finite number of periods, rather than for each and every period ad infinitum.

to human capital, w^*_{t+1}, that are higher than at home: $w^*_{t+1} > w_{t+1}$. Taking this prospect of migration in the second period of his life into consideration, the individual's maximization problem is

$$\max_{\tilde{e}_t} E(\tilde{c}_{t+1}) = \overline{w}_{t+1}\tilde{h}_{t+1} - R \cdot \tilde{e}_t$$

subject to (1) and $\tilde{c}_{t+1} \geq 0$, where $\overline{w}_{t+1} = (1-p)w_{t+1} + pw^*_{t+1} > w_{t+1} \geq A$, and where a tilde represents the level of a variable under migration. We will momentarily add assumptions in order to ensure that this optimization problem has an interior solution at which the non-negativity constraint on consumption is not binding. Proceeding analogously to (4)–(6), we get

$$\tilde{e}_t = (\alpha\overline{w}_{t+1}/R)^{1/(1-\alpha)}\tilde{h}_t^{\beta/(1-\alpha)}, \tag{10}$$

$$E(\tilde{c}_{t+1}) = (1-\alpha)\overline{w}_{t+1}{}^{1/(1-\alpha)}(\alpha/R)^{\alpha/(1-\alpha)}\tilde{h}_t^{\beta/(1-\alpha)}, \tag{11}$$

and

$$\tilde{h}_{t+1} = (\alpha\overline{w}_{t+1}/R)^{\alpha/(1-\alpha)}\tilde{h}_t^{\beta/(1-\alpha)}. \tag{12}$$

A comparison of (10) with (4), and of (12) with (6) unravels the *static* inducement effect of migration on human capital formation, studied in Stark and Wang (2002): the higher expected wage rate yielded by the prospect of migration induces individuals to invest more in human capital formation $(\tilde{e}_t > e_t)$ and, correspondingly, the level of human capital formed in the economy is higher $\left(\tilde{h}_{t+1} > h_{t+1}\right)$. We next show that the prospect of migration, which translates into an improvement of total productivity, can give rise to yet another, *dynamic* gain in the long run.

For the sake of tractability and to concentrate on essentials, we will not model the determination of the foreign wage rate but assume, instead, that in each period t the foreign wage rate is equal to a multiple of the domestic wage rate: $w^*_t = \kappa w_t$, where $\kappa > 1$ is a constant. Thus, under migration, the expected returns to human capital are given by $\overline{w}_{t+1} = (1-p)w_{t+1} + pw^*_{t+1} = \overline{\kappa}w_{t+1}$, where $\overline{\kappa} \equiv 1 - p + p\kappa > 1$. Using (10) and (12) we find that for individuals who do not migrate

$$\tilde{c}_{t+1} = (\alpha\overline{w}_{t+1}/R)^{\alpha/(1-\alpha)}\tilde{h}_t^{\beta/(1-\alpha)}(w_{t+1} - \alpha\overline{w}_{t+1}) \tag{13}$$

and for individuals who migrate

$$\tilde{c}_{t+1} = (\alpha\overline{w}_{t+1}/R)^{\alpha/(1-\alpha)}\tilde{h}_t^{\beta/(1-\alpha)}(w^*_{t+1} - \alpha\overline{w}_{t+1}). \tag{14}$$

It follows that $\tilde{c}_{t+1} \geq 0$ holds with probability 1 provided that $w_{t+1} - \alpha\overline{w}_{t+1} = (1-\alpha\overline{\kappa})w_{t+1} \geq 0$, which, in turn, is satisfied as long as $1 - \alpha\overline{\kappa} \geq 0$. This assumption, which means that either the foreign wage is not too high as compared to the domestic wage (that is, κ is sufficiently close to 1) or that the probability of migration is sufficiently small (that is, p is sufficiently close to 0), will therefore be maintained for the remainder of our analysis. We collect our assumptions on p and κ as follows.

Assumption 3. The parameters p and κ satisfy $0 < p < 1$, $\kappa > 1$, and $1 - p + p\kappa \leq 1/\alpha$.

Because $\overline{w}_{t+1} = \overline{\kappa}w_{t+1}$, it follows by combining (6), (7), and (12), that

$$\tilde{h}_{t+1} = \overline{\kappa}^{\alpha/(1-\alpha)}H\left(\tilde{h}_t\right) = \begin{cases} \left(\frac{\alpha\overline{\kappa}A}{R}\right)^{\alpha/(1-\alpha)}\tilde{h}_t^{\beta/(1-\alpha)} & \text{for } \tilde{h}_t < \overline{h}; \\ \left(\frac{\alpha\overline{\kappa}A}{R\overline{h}^{\rho}}\right)^{\alpha/(1-\alpha)}\tilde{h}_t^{(\alpha\rho+\beta)/(1-\alpha)} & \text{for } \tilde{h}_t \geq \overline{h}. \end{cases} \tag{15}$$

We denote the first and second segments of the human capital transition dynamics in (15) by

$$\tilde{\phi}\left(\tilde{h}_t\right) \equiv \left(\frac{\alpha\overline{\kappa}A}{R}\right)^{\alpha/(1-\alpha)}\tilde{h}_t^{\beta/(1-\alpha)} = \overline{\kappa}^{\alpha/(1-\alpha)}\phi\left(\tilde{h}_t\right)$$

and

$$\tilde{\varphi}\left(\tilde{h}_t\right) \equiv \left(\frac{\alpha\overline{\kappa}A}{R\overline{h}^{\rho}}\right)^{\alpha/(1-\alpha)}\tilde{h}_t^{(\alpha\rho+\beta)/(1-\alpha)} = \overline{\kappa}^{\alpha/(1-\alpha)}\varphi\left(\tilde{h}_t\right),$$

respectively. The transition function of human capital under migration is represented by an upward shift ($\overline{\kappa} > 1$) of the transition function of human capital without migration. Consequently, the dynamic properties of human capital formation in the economy with migration crucially depend on the size of this shift, as determined by the magnitude of the parameter $\overline{\kappa}$. Similarly to the case of the closed economy, we have that $\tilde{\phi}(\cdot)$ is a concave function and that all the solutions of $h_{t+1} = \tilde{\phi}(h_t)$ converge monotonically to the unique steady state at

$$\tilde{h}^* = (\alpha\overline{\kappa}A/R)^{\alpha/(1-\alpha-\beta)}. \tag{16}$$

On the other hand, the function $\tilde{\varphi}(\cdot)$ can display various shapes, depending on the parameter η. As long as $\eta \neq 1$, $\tilde{\varphi}(\cdot)$ yields a unique steady state at

$$\tilde{h}^{**} = \left(\alpha\overline{\kappa}A/R\overline{h}^{\rho}\right)^{\alpha/(1-\eta)}, \tag{17}$$

which is stable under the dynamics $h_{t+1} = \tilde{\varphi}(h_t)$ if $\eta < 1$, and unstable if $\eta > 1$. Once again, for the sake of fleshing out our main argument, we resort to an assumption.

Assumption 4. It holds that $(\alpha\overline{\kappa}A/R)^{\alpha/(1-\alpha-\beta)} > \overline{h}$.

Under Assumption 4, starting from any initial level of human capital $h_0 < \overline{h}$, the static inducement effect as proxied by $\overline{\kappa}$ is strong enough to lift the level of human capital above the threshold level $\overline{h}$ along the transition function $\tilde{\phi}(\cdot)$. Then, due to the intertemporal externality, the amplified productivity factor will further alter the future trajectory of the human capital accumulation dynamics and thereupon, the growth path of the economy. Because the average level of human capital will cross the threshold level of human capital $\overline{h}$ at some point in time, we may assume, without loss of generality, that the transition dynamics of human capital is given by $\tilde{\varphi}(\cdot)$ in every period, as per the second segment of (15) for $\tilde{h}_t \geq \overline{h}$. The following result then obtains.

Lemma 2. The level of human capital formed by a young individual under migration will grow without bounds iff $\eta = \alpha(1+\rho) + \beta \geq 1$, and it will converge to the constant level $\tilde{h}^{**}$ in (17), where $\tilde{h}^{**} > \overline{h}$, iff $\eta < 1$.

Proof. Let us define $M = (\alpha\overline{\kappa}A/R)^{\alpha/(1-\alpha)}$. Then we can rewrite $\tilde{\phi}(\cdot)$ as $\tilde{\phi}\left(\tilde{h}_t\right) = M\tilde{h}_t^{\beta/(1-\alpha)}$, and $\tilde{\varphi}(\cdot)$ as $\tilde{\varphi}\left(\tilde{h}_t\right) = \tilde{\phi}\left(\overline{h}\right) \cdot \left(\tilde{h}_t/\overline{h}\right)^{(\alpha\rho+\beta)/(1-\alpha)}$. Because $\tilde{\phi}\left(\tilde{h}_t\right)$ is a strictly concave function with a unique fixed point at $\tilde{h}^*$ as specified in (16), Assumption 4 implies that $\tilde{h}^* > \overline{h}$ and hence, $\tilde{\phi}\left(\overline{h}\right) > \overline{h}$. Thus, we can find $\varepsilon > 0$ such that $\tilde{\phi}\left(\overline{h}\right) > (1+\varepsilon)\overline{h}$. In the case where $\eta \geq 1$ we have $(\alpha\rho + \beta)/(1-\alpha) \geq 1$ and it follows therefore from the preceding results that

$$\tilde{h}_{t+1} = \tilde{\varphi}\left(\tilde{h}_t\right) \geq \tilde{\phi}\left(\overline{h}\right)\left(\tilde{h}_t/\overline{h}\right) > (1+\varepsilon)\tilde{h}_t$$

holds for all $\tilde{h}_t > \overline{h}$. This proves that whenever $\eta \geq 1$ and the human capital stock has grown beyond the threshold level $\overline{h}$, it continues to grow without bound. However, when $\eta < 1$, $\tilde{\varphi}\left(\tilde{h}_t\right)$ is a strictly concave function that intersects the 45 degree line to the right of $\overline{h}$ at $\tilde{h}^{**}$ as specified in (17) (because $\tilde{\varphi}\left(\overline{h}\right) = \tilde{\phi}\left(\overline{h}\right) > \overline{h}$). It is easily seen that in this case $\tilde{h}_t$ must converge to that steady state value. □

The human capital dynamics under migration, as given in (15), is illustrated in Fig. 2.

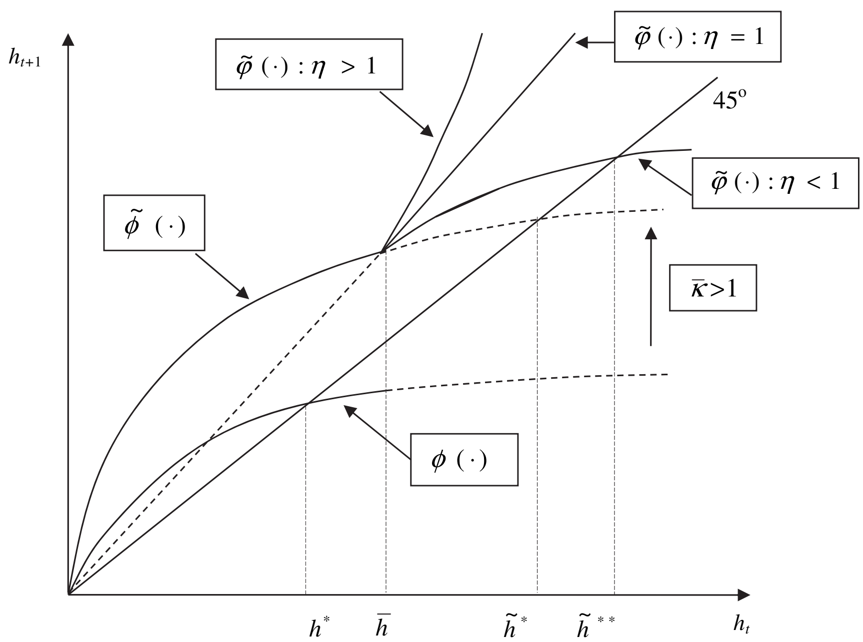

Fig. 2. The phase diagram of the human capital dynamics under migration.

The impact of the intertemporal externality of human capital on productivity when the level of human capital is above the threshold level of human capital $\bar{h}$ (an impact captured by the parameter ρ) cannot be realized in the absence of migration in the benchmark economy because the human capital dynamics is trapped below the threshold level $\bar{h}$. The presence of the prospect of migration thus unlocks this potential intertemporal externality by lifting the human capital stock above the threshold level $\bar{h}$. Lemma 2 highlights the possibility that instead of converging to a constant steady state level of human capital, the human capital formed by young individuals will grow unboundedly generation after generation as and when the pull of the prospect of migration is powerful enough. On the other hand, if the effect of the intertemporal externality of human capital on productivity exists yet is not strong enough (that is, if $\rho > 0$ but $\alpha(1+\rho)+\beta<1$), the prospect of migration leads only to a level effect on human capital formation in the long run $\left(\tilde{h}^{**} > h^*\right)$.

It is noteworthy that Lemma 2, and indeed the very gist of our argument, apply even when the prospect of migration avails itself not for each and every generation ad infinitum but, rather, for a limited period of time that is nonetheless long enough to lift the level of human capital through the accumulation process of $\tilde{h}_{t+1} = \tilde{\varphi}\left(\tilde{h}_t\right)$ sufficiently high above the threshold level $\bar{h}$ and thereby unleashes the power of the intertemporal externality of human capital. To elucidate this point, suppose that the door of migration is shut, say as of some period t, after the average level of human capital has risen above the level $\bar{h}$. As long as $\tilde{h}_t > \bar{h}$ holds, the transition dynamics of human capital *without* the prospect of migration is given by

$$\tilde{h}_{t+1} = \varphi\left(\tilde{h}_t\right) = \left(\frac{\alpha A}{R\bar{h}^{\rho}}\right)^{\alpha/(1-\alpha)} \tilde{h}_t^{(\alpha\rho+\beta)/(1-\alpha)}. \tag{18}$$

It is easy to see that the human capital level in (18) thereupon rises without bounds iff $\eta = \alpha(1+\rho)+\beta > 1$ and $\tilde{h}_t > \left(\alpha A/R\bar{h}^{\rho}\right)^{\alpha/(1-\eta)}$.[8] On the other hand, if $\eta \le 1$ or $\tilde{h}_t < \left(\alpha A/R\bar{h}^{\rho}\right)^{\alpha/(1-\eta)}$, then $\tilde{h}_t$ eventually falls below $\bar{h}$, from which point onwards the human capital dynamics is again described by $\phi(\cdot)$. In that case it follows that $\tilde{h}_t$ converges to h^* as given in (8). For ease of exposition, we consider in the remainder of our analysis the case in which the prospect of migration is present in each and every time period.

We next show that, under the assumption $\eta > 1$, migration will deliver a growth gain in the long run if the intertemporal externality of human capital formation is large enough.

Proposition 1. If $\eta > 1$, then it holds for all sufficiently large t that $\tilde{g}_t > g^* = n$.

Proof. Because the human capital level will surpass the threshold level $\bar{h}$ at some point in time, we assume without loss of generality that $A_{t+1} = f\left(\tilde{h}_t\right) = \left(A/\bar{h}^{\rho}\right)\tilde{h}_t^{\rho}$ applies in every period. Making use of $\tilde{h}_{t+1} = \tilde{\varphi}\left(\tilde{h}_t\right)$ for all t and recalling that $\eta = \alpha(1+\rho)+\beta$, the growth rate under migration is then given by

$$\begin{aligned}
\tilde{g}_t &= \tilde{Y}_{t+1}/\tilde{Y}_t = A_{t+1}\tilde{L}_{t+1}\tilde{h}_{t+1}/\left(A_t\tilde{L}_t\tilde{h}_t\right) = (1-p)n(A_{t+1}/A_t)\left(\tilde{h}_{t+1}/\tilde{h}_t\right) = (1-p)n\left(\tilde{h}_t/\tilde{h}_{t-1}\right)^{\rho}\left(\tilde{h}_{t+1}/\tilde{h}_t\right) \\
&= (1-p)n\left(\alpha\bar{\kappa}A/R\bar{h}^{\rho}\right)^{\alpha\rho/(1-\alpha)}\tilde{h}_{t-1}^{(\eta-1)\rho/(1-\alpha)}\left(\alpha\bar{\kappa}A/R\bar{h}^{\rho}\right)^{\alpha/(1-\alpha)}\tilde{h}_t^{(\eta-1)/(1-\alpha)} \\
&= (1-p)n\left(\alpha\bar{\kappa}A/R\bar{h}^{\rho}\right)^{\alpha(1+\rho)/(1-\alpha)}\tilde{h}_{t-1}^{(\eta-1)\rho/(1-\alpha)}\left(\alpha\bar{\kappa}A/R\bar{h}^{\rho}\right)^{\alpha(\eta-1)/(1-\alpha)^2}\tilde{h}_{t-1}^{(\alpha\rho+\beta)(\eta-1)/(1-\alpha)^2} \\
&= (1-p)n\left(\alpha\bar{\kappa}A/R\bar{h}^{\rho}\right)^{\alpha(\beta+\rho)/(1-\alpha)^2}\tilde{h}_{t-1}^{(\eta-1)(\beta+\rho)/(1-\alpha)^2}.
\end{aligned}$$

If $\eta > 1$, because $\tilde{h}_{t-1}$ grows without bounds (cf. Lemma 2), and because the exponent of the last term is strictly positive, it is clear that $\tilde{g}_t > g^* = n$ holds for all large enough t. □

It is worth noting that the dynamic growth gain alluded to in Proposition 1 would not have been possible in the absence of a strong enough intertemporal externality of the formation of human capital (that is, if it were the case that $\rho = 0$, or if it were the case that $\rho > 0$ but $\eta < 1$). Absent a sufficiently strong intertemporal externality, the human capital formed by an individual would have converged to the steady state level $\tilde{h}^{**}$ and, consequently, the growth rate would have converged to $\tilde{g}^* = (1-p)n < g^* = n$. Put differently, the static skill gain from the "inducement effect" $\left(\tilde{h}_t > h_t\right)$ notwithstanding, if the intertemporal externality of the human capital formation were absent or present but weak enough, the prospect of migration would have actually led to a lower growth rate in the long run. It is also worth adding that the long-run growth gain achieved by the economy under the stipulated migration regime

[8] This follows from the fact that when $\eta > 1$, the dynamics described by (18) has a unique unstable steady state at $\left(\alpha A/R\bar{h}^{\rho}\right)^{\alpha/(1-\eta)}$; see (9). The condition $\eta > 1$ is slightly stronger than the condition needed to guarantee an unbounded rise in human capital formation when the prospect of migration is present in each and every period; see Lemma 2.

is in terms of aggregate output, despite of the constant depletion of the economy's better educated labor force due to migration. Therefore, in terms of per-capita output, the long-run growth gain would be even larger.

A special case

When $\eta = \alpha(1+\rho)+\beta = 1$, both the average level of human capital and the total output of the economy, Y_t, grow in the long run at constant rates. As a matter of fact, it follows from the transition function $\tilde{\varphi}(\cdot)$ that in this case

$$\tilde{h}_{t+1} = \left(\alpha\overline{\kappa}A/R\overline{h}^{\rho}\right)^{\alpha/(1-\alpha)}\tilde{h}_t = \Omega\cdot\tilde{h}_t,$$

where $\Omega \equiv (\alpha\overline{\kappa}A/R)^{\alpha/(1-\alpha)}\overline{h}^{-(1-\alpha-\beta)/(1-\alpha)} > 1$ holds by Assumption 4, and

$$\tilde{g}_t = \tilde{g}^* = (1-p)n\left(\alpha\overline{\kappa}A/R\overline{h}^{\rho}\right)^{(1-\beta)/(1-\alpha)} = (1-p)n\Omega^{(1-\beta)/\alpha}.$$

Then, a long-run growth gain $\tilde{g}^* > g^* = n$ obtains if and only if $(1-p)\Omega^{(1-\beta)/\alpha} > 1$ (which, because $\Omega > 1$, holds for sufficiently small values of p).

In the general case, the growth rate can increase without bounds because $\tilde{h}_t$ does. In the special case, however, the growth rate converges to a constant, so the economy exhibits a long-run balanced growth path along which output grows at a constant rate. Under the right parameter configurations, more precisely if $(1-p)(\alpha\overline{\kappa}A/R)^{(1-\beta)/(1-\alpha)}\overline{h}^{-(1-\beta)(1-\alpha-\beta)/[\alpha(1-\alpha)]} > 1$, this balanced growth rate will be higher than the growth rate in the non-migration economy (hence the term "growth gain").[9]

4. Welfare repercussions

Measuring welfare by the consumption level of a representative individual who stays in the home country, we can show that the prospect of migration can indeed confer a welfare gain in the long run once the growth gain is operative. This welfare comparison assumes away but does not neglect the migrants. Individuals who migrate and subject their human capital to the returns to human capital of the foreign country, which are higher than those in the home country, are strictly better off than the individuals who stay behind. Consequently, a welfare gain for the individuals who stay behind implies a welfare gain for all the home country individuals.

To prove this claim, we first recall from Eq. (5) that the per capita consumption level in the closed economy is given by

$$c_{t+1} = (1-\alpha)A^{1/(1-\alpha)}(\alpha/R)^{\alpha/(1-\alpha)}h_t^{\beta/(1-\alpha)},$$

where we have drawn on the property that $w_{t+1} = A$ holds for all $h_t < \overline{h}$. From Lemma 1 and Eq. (8) it follows that this consumption level converges to the steady state value

$$c^* = (1-\alpha)A^{(1-\beta)/(1-\alpha-\beta)}(\alpha/R)^{\alpha/(1-\alpha-\beta)}. \tag{19}$$

Analogously, from (13) and $\overline{w}_{t+1} = \overline{\kappa}w_{t+1}$, the per-capita consumption level of a representative individual who ends up staying in the home country is

$$\tilde{c}_{t+1} = (1-\alpha\overline{\kappa})w_{t+1}{}^{1/(1-\alpha)}(\alpha\overline{\kappa}/R)^{\alpha/(1-\alpha)}\ \tilde{h}_t{}^{\beta/(1-\alpha)}.$$

Because the level of human capital in the presence of the migration prospect will cross the threshold level $\overline{h}$ at some point in time, we can assume, without loss of generality, that $w_{t+1} = \left(A/\overline{h}^{\rho}\right)\tilde{h}_t^{\rho}$, and hence we have that

$$\tilde{c}_{t+1} = (1-\alpha\overline{\kappa})A^{1/(1-\alpha)}\overline{h}^{-\rho/(1-\alpha)}(\alpha\overline{\kappa}/R)^{\alpha/(1-\alpha)}\tilde{h}_t{}^{(\beta+\rho)/(1-\alpha)}. \tag{20}$$

Under the conditions stipulated in Proposition 1, the human capital level $\tilde{h}_t$ will grow without bounds. This implies that, as long as $\alpha\overline{\kappa} < 1$, which is just slightly stronger than what we have assumed in order to ensure that all the individuals can repay their loans, the per capita consumption level of an individual who remains in the home country will grow without bounds as well. Therefore, in comparison to the non-migration economy in which consumption converges to a constant steady state level, the individuals staying in the home country will experience a welfare gain in terms of per capita consumption, unless $\alpha\overline{\kappa} = 1$ (in which case the non-negativity constraint on consumption becomes binding). This welfare gain is conferred by the previously discussed long-run growth gain.

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[9] However, because the growth rate of *per-capita* output along the balanced growth path is given by $\Omega^{(1-\beta)/\alpha} > 1$, a growth gain in terms of *per-capita* output is always guaranteed in this case.

The assumption in Proposition 1 that guarantees a long-run growth gain is however much stronger than what is needed for a welfare gain to accrue under migration. In fact, even if a growth gain is absent, for example if the intertemporal externality of human capital that emanates from migration is either too small (as in the case when $\eta = \alpha(1+\rho)+\beta < 1$) or even nonexistent (as in the case when the static inducement effect as measured by $\overline{\kappa}$ is too weak to render the intertemporal externality effective), a welfare gain is still possible under a "controlled" migration. This we illustrate in the remainder of this section under the assumption $\eta < 1$.

Suppose, first, that the probability of migration p is so small (and, hence, $\overline{\kappa}$ is so close to 1) that Assumption 4 fails to hold. In this case, the human capital formed by individuals in the presence of migration, $\tilde{h}_t$, converges to $\tilde{h}^*$ as given in (16). Because this value is smaller than $\overline{h}$, the intertemporal externality of human capital formation will never become operative, a situation that is essentially equivalent to setting $\rho = 0$ in our model. Hence, by substituting (16) into (20) and setting $\rho = 0$, we see that

$$\tilde{c}^* = (1-\alpha\overline{\kappa})A^{(1-\beta)/(1-\alpha-\beta)}(\alpha\overline{\kappa}/R)^{\alpha/(1-\alpha-\beta)}. \tag{21}$$

Combining (19) and (21) it follows that

$$\tilde{c}^*/c^* = G(\overline{\kappa}) \equiv \left[(1-\alpha\overline{\kappa})/(1-\alpha)\right]\overline{\kappa}^{\alpha/(1-\alpha-\beta)}. \tag{22}$$

We now note that $G(1) = 1$, $G'(\overline{\kappa}) > 0$ for $\overline{\kappa} < 1/(1-\beta)$, and $G'(\overline{\kappa}) < 0$ for $\overline{\kappa} > 1/(1-\beta)$. These observations imply that there exists a $\overline{\kappa}^* > 1/(1-\beta)$ such that $G(\overline{\kappa}) > 1$ holds whenever $1 < \overline{\kappa} < \overline{\kappa}^*$ which, by the very definition of $\overline{\kappa}$, is satisfied when p is small enough.

We next consider the case in which p is small enough so that $\overline{\kappa} < \overline{\kappa}^*$ holds, but is still high enough so that Assumption 4 holds or, equivalently, that $\overline{\kappa} > \overline{\kappa}' \equiv (\alpha A/R)^{-1}\overline{h}^{(1-\alpha-\beta)/\alpha}$ is satisfied (recall that from Assumption 2 it follows that $\overline{\kappa}' > 1$).[10] Under Assumption 4, the human capital formed by individuals in the presence of migration, $\tilde{h}_t$, converges to $\tilde{h}^{**}$ in (17). Together with (20) this implies that the consumption level of the individuals who do not migrate, $\tilde{c}_t$, converges to the steady state level

$$\tilde{c}^* = (1-\alpha\overline{\kappa})A^{(1-\beta)/(1-\eta)}\overline{h}^{-(1-\beta)\rho/(1-\eta)}(\alpha\overline{\kappa}/R)^{\alpha(1+\rho)/(1-\eta)}. \tag{23}$$

From (19) and (23) we obtain

$$\tilde{c}^*/c^* = \left[(1-\alpha\overline{\kappa})/(1-\alpha)\right](\alpha A/R)^{\gamma}\overline{\kappa}^{\alpha(1+\rho)/(1-\eta)}\overline{h}^{-(1-\beta)\rho/(1-\eta)} \tag{24}$$

where $\gamma = \alpha\rho(1-\beta)/[(1-\alpha-\beta)(1-\eta)]$. Using (23) and (16), we can rewrite the condition $\tilde{c}^* > c^*$ as $G(\overline{\kappa}) > \left(\overline{h}/\tilde{h}^*\right)^{(1-\beta)\rho/(1-\eta)}$. Because of $\tilde{h}^* > \overline{h}$ (due to Assumption 4) and $\eta < 1$, this inequality is satisfied whenever its left-hand side, $G(\overline{\kappa})$, is strictly larger than 1. Because we have already shown that $G(\overline{\kappa}) > 1$ holds whenever $1 < \overline{\kappa} < \overline{\kappa}^*$, it follows that $\tilde{c}^* > c^*$ holds for all p such that $\overline{\kappa}' < \overline{\kappa} < \overline{\kappa}^*$ is satisfied.

This last result illustrates that even if the intertemporal externality of human capital is not large enough to foster a long-run growth gain, the long-run level effect on human capital formation can still confer a welfare gain upon the individuals who stay behind in the home country. The fact that this finding holds even when the "pull" of the inducement effect (as measured by $\overline{\kappa}$) is too small so that it fails to trigger the intertemporal externality altogether suggests that the result of a welfare gain from migration is quite robust.

5. The case of heterogeneous individuals

It might be argued that so far our analysis has omitted an important consideration, namely that often it is the better educated individuals who succeed in getting job offers from developed countries. If such is the case, migration would impose a negative externality on non-migrants as it causes the average level of human capital to decline. Obviously, for this perspective to be studied, a model that allows for heterogeneity is needed. In this section we discuss a variant of our model that is amenable to such a study. We assume that individuals can be of two types: low-skill and high-skill. The difference in types is reflected by the high-skill individuals having a higher productivity in acquiring human capital than the low-skill individuals. Furthermore, we assume that only the high-skill individuals, who in equilibrium form more human capital, are able to migrate. Although this is a strong assumption, it is the very assumption that generates the strongest negative externality through migration. We seek to demonstrate that even in such a setting, the incentives created by the prospect of migrating to a high-income country may result in a growth gain, as well as in a consequent welfare gain.

Suppose that in each generation a fraction $\delta \in (0,1)$ of individuals has low skills, whereas the complementary fraction $1-\delta$ has high skills. Individuals of the high skill type have the same production function for human capital as in (1). Individuals of the low skill type have the human capital production function

$$h_{t+1} = \lambda e_t^{\ \alpha} h_t^{\ \beta}$$

[10] From $\overline{\kappa}^* > 1/(1-\beta)$ and $\overline{\kappa}' = (\alpha A/R)^{-1}\overline{h}^{(1-\alpha-\beta)/\alpha}$, it is easy to see that the parameter space for which $\overline{\kappa}' < \overline{\kappa}^*$ holds is nonempty.

where $0 < \lambda < 1$. Individuals know their types when in the first period of their life they make their human capital acquisition decisions. This implies that those who have low skills (and, by assumption, are aware of that) correctly anticipate that they will not be able to migrate. Denoting again the unconditional probability of an individual migrating by p, we would have to assume that $p \leq 1-\delta$. Furthermore, it follows that the conditional probability of a high-skill individual migrating is $p^* = p/(1-\delta)$, whereas the conditional probability of a low-skill individual migrating is zero.

We start with the case of no migration. Analogously to Section 2, absent a prospect of migration, a high-skill individual of generation t chooses to form the human capital level $h_{t+1} = (\alpha w_{t+1}/R)^{\alpha/(1-\alpha)} H_t^{\beta/(1-\alpha)}$, where H_t is the average level of human capital in the home country in period t. On the other hand, low-skill individuals choose $h_{t+1} = (\alpha\lambda w_{t+1}/R)^{\alpha/(1-\alpha)} H_t^{\beta/(1-\alpha)}$. Therefore, the evolution of the average level of human capital is given by

$$H_{t+1} = \begin{cases} \Lambda\left(\dfrac{\alpha A}{R}\right)^{\alpha/(1-\alpha)} H_t^{\beta/(1-\alpha)} & \text{for } H_t < \overline{h}; \\ \Lambda\left(\dfrac{\alpha A}{R\overline{h}^{\rho}}\right)^{\alpha/(1-\alpha)} H_t^{(\alpha\rho+\beta)/(1-\alpha)} & \text{for } H_t \geq \overline{h}, \end{cases} \tag{25}$$

where $\Lambda = 1-\delta+\delta\lambda^{\alpha/(1-\alpha)} < 1$. As in the case of homogeneous individuals, we assume that the threshold level of human capital at which the intertemporal externality sets in is higher than the steady state value rendered by (25) without the externality (that is, the second line of (25)).[11] Formally, this means that $\overline{h} > \Lambda^{(1-\alpha)/(1-\alpha-\beta)}(\alpha A/R)^{\alpha/(1-\alpha-\beta)}$.

We now allow for migration. With regard to high-skill individuals, Eqs. (10)–(14) remain valid without a change if we merely redefine $\overline{w}_{t+1}$ as $\overline{w}_{t+1} = (1-p^*)w_{t+1} + p^* w^*_{t+1}$ and reformulate Assumption 3 as $\kappa^* \equiv 1-p^*+p^*\kappa \leq 1/\alpha$. For low-skill individuals, however, we have to reckon that they are aware of the fact that they will not be able to migrate. Hence, they know that they will be earning the domestic wage w_{t+1} for sure and, therefore, they behave as in the no-migration case, choosing the human capital level $h_{t+1} = (\alpha\lambda w_{t+1}/R)^{\alpha/(1-\alpha)} H_t^{\beta/(1-\alpha)}$. Combining these observations, we find that the average level of human capital evolves according to

$$\begin{aligned} \tilde{H}_{t+1} &= (1-\delta)\left(\alpha\overline{w}_{t+1}/R\right)^{\alpha/(1-\alpha)} \tilde{H}_t^{\beta/(1-\alpha)} + \delta\left(\alpha\lambda w_{t+1}/R\right)^{\alpha/(1-\alpha)} \tilde{H}_t^{\beta/(1-\alpha)} \\ &= \Lambda^*\left(\alpha w_{t+1}/R\right)^{\alpha/(1-\alpha)} \tilde{H}_t^{\beta/(1-\alpha)}, \end{aligned}$$

where $\Lambda^* = (1-\delta)(\kappa^*)^{\alpha/(1-\alpha)} + \delta\lambda^{\alpha/(1-\alpha)}$. We note that $\kappa > 1$ implies that $\kappa^* > 1$ which, in turn, implies $\Lambda^* > \Lambda$. Thus, the equation corresponding to (15) now reads as

$$\tilde{H}_{t+1} = \begin{cases} \Lambda^*\left(\dfrac{\alpha A}{R}\right)^{\alpha/(1-\alpha)} \tilde{H}_t^{\beta/(1-\alpha)} & \text{for } \tilde{H}_t < \overline{h}; \\ \Lambda^*\left(\dfrac{\alpha A}{R\overline{h}^{\rho}}\right)^{\alpha/(1-\alpha)} \tilde{H}_t^{(\alpha\rho+\beta)/(1-\alpha)} & \text{for } \tilde{H}_t \geq \overline{h}, \end{cases} \tag{26}$$

and Assumption 4 translates into $(\Lambda^*)^{(1-\alpha)/(1-\alpha-\beta)}(\alpha A/R)^{\alpha/(1-\alpha-\beta)} > \overline{h}$.

From a comparison of Eqs. (25) and (26) with Eqs. (7) and (15), respectively, it becomes apparent that introducing heterogeneity does not qualitatively change any of the results of Section 3. In particular, under the appropriate reformulation of Assumptions 2–4 as mentioned above, results analogous to Lemma 2 and to Proposition 1 can be obtained. This implies that the growth gain alluded to in the title of the paper is not a consequence of the assumption of a homogeneous workforce. Finally, as we have shown in Section 4, at least in the case where the growth gain is operative, that is, at least in the case where the intertemporal externality is strong enough, this growth gain confers a welfare gain also upon all the individuals who do not migrate. It is straightforward to verify that this result carries over to the case of heterogeneous individuals.

6. Conclusion

Complementing Stark and Wang (2002), we have provided conditions under which, in a dynamic setting with sufficiently large intertemporal human capital externalities, the gains to a developing country that accrue from the inducement effect of the prospect of migration for the formation of human capital include long-term economic growth. The growth gains are conferred when the migration door is opened period by period, and even if it is shut after a finite number of periods. In line with the results of Stark and Wang (2002) for the static, one period analysis, we showed, in the dynamic setting studied in the current paper, that there are welfare gains to all, migrants and non-migrants alike, provided that the probability of migration is relatively small. Perhaps the most powerful message of the current paper is that when the intertemporal spillover effect of the current human capital on future productivity is sufficiently strong, the response of individuals to the prospect of migration is a catalyst of long-run growth. Furthermore, when the spillover effect of the prospect of migration confers a long-run growth gain in the home country, even a temporary opening up to migration can lead to long-lasting beneficial consequences, both for growth and for welfare.

[11] For a comparison with the case of homogeneous individuals, see Assumption 2.

As noted in the introduction, the main idea of our earlier work is that the prospect of migration from a developing country to a developed country changes not only the *set of opportunities* but also the *structure of the incentives* that individuals face: higher prospective returns to human capital in a developed country induce more human capital formation in a developing country. In the present paper we have gone beyond our original idea. We have argued that the prospect of migration reshapes the *environment for intertemporal human capital formation* in the developing country: not only does it strengthen the incentives to form human capital in the present; it also makes the prevailing human capital infrastructure for the subsequent formation of human capital more hospitable and more inviting.

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Appendix A. Proof of Lemma 1

Assumption 2 is equivalent to $\overline{h} > \phi(\overline{h}) = \varphi(\overline{h})$, that is, the graph of (7) at $\overline{h}$ is below the 45 degree line. Depending on the size of the intertemporal externality, ρ, and hence on the magnitude of η, there are three possibilities (see Fig. 1).

First, if $\eta < 1$ (when the intertemporal externality is relatively small), $\varphi(\cdot)$ is concave and has a unique stable steady state. Denoting the steady state in (9) in this case of $\eta < 1$ as h_1^{**}, it follows from $\overline{h} > (\alpha A/R)^{\alpha/(1-\alpha-\beta)} = h^*$ that $h_1^{**} < h^* < \overline{h}$. Thus, the transition function $H(h_t) = \phi(h_t)$ for $h_t < \overline{h}$ observes $\overline{h} > \phi(\overline{h}) = \varphi(\overline{h})$; and $H(h_t) = \varphi(h_t)$ for $h_t \geq \overline{h}$ is concave and never intersects the 45 degree line again (because $h_1^{**} < h^* < \overline{h}$). Therefore, $h_{t+1} = H(h_t)$ has a unique stable steady state at h^* over the entire domain.

Second, if $\eta = 1$ (the special case), $\varphi(\cdot)$ is linear with a slope $\left(\alpha A/R\overline{h}^{\rho}\right)^{\alpha/(1-\alpha)}$ that is less than 1 because $\overline{h} > (\alpha A/R)^{\alpha/(1-\alpha-\beta)}$. Thus, the transition function $H(h_t) = \phi(h_t)$ for $h_t < \overline{h}$ observes $\overline{h} > \phi(\overline{h}) = \varphi(\overline{h})$; and $H(h_t) = \varphi(h_t)$ for $h_t \geq \overline{h}$ is linear with a slope of less than 1. Therefore, $h_{t+1} = H(h_t)$ has a unique stable steady state at h^* over the entire domain.

Third, if $\eta > 1$ (when the intertemporal externality is relatively large), $\varphi(\cdot)$ is convex and the unique steady state is unstable. Denoting the steady state in (9) in this case of $\eta > 1$ as h_2^{**}, it follows from $\overline{h} > (\alpha A/R)^{\alpha/(1-\alpha-\beta)}$ that $h_2^{**} > \overline{h} > h^*$. Thus, the transition function $H(h_t) = \phi(h_t)$ for $h_t < \overline{h}$ satisfies $\overline{h} > \phi(\overline{h}) = \varphi(\overline{h})$; and $H(h_t) = \varphi(h_t)$ for $h_t \geq \overline{h}$ is convex and intersects the 45 degree line from below at h_2^{**}. Therefore, $h_{t+1} = H(h_t)$ has a stable steady state at h^* and an unstable steady state at h_2^{**} as determined in (9). It then follows that h_t converges to h^* as long as $h_0 < h_2^{**} = \left(\alpha A/R\overline{h}^{\rho}\right)^{\alpha/(1-\eta)}$.

The preceding analyses of the three possibilities complete the proof. □

Appendix B. The Case of a Relatively Low Threshold Level of Human Capital

B.1. The human capital dynamics in the closed economy

In the case $\overline{h} < (\alpha A/R)^{\alpha/(1-\alpha-\beta)} = h^*$, the threshold level of human capital for the intertemporal externality is relatively low, and Assumption 2 is violated. The characterization of the human capital dynamics can once again be divided into three sub-cases, depending on the size of the intertemporal externality, ρ. Similar to the proof of Lemma 1, it is easy to show that the human capital dynamics is characterized as follows (and as illustrated in Fig. 3).

Sub-case B.1.1. If $\eta < 1$ (the intertemporal externality is small)

In this sub-case, the transition function $H(h_t) = \phi(h_t)$ is above the 45 degree line for $h_t < \overline{h}$, and for $h_t \geq \overline{h}$, $H(h_t) = \varphi(h_t)$ is concave and intersects the 45 degree line from above at h_1^{**}, which satisfies $h_1^{**} > h^* > \overline{h}$. Therefore, $h_{t+1} = H(h_t)$ has a unique stable steady state at h_1^{**} over the entire domain.

Sub-case B.1.2. If $\eta > 1$ (the intertemporal externality is large)

In this sub-case, the transition function $H(h_t) = \phi(h_t)$ is above the 45 degree line for $h_t < \overline{h}$, and for $h_t \geq \overline{h}$, $H(h_t) = \varphi(h_t)$ is convex and never intersects the 45 degree line. Therefore, $H(h_t)$ does not have an interior steady state.

Sub-case B.1.3. If $\eta = 1$ (the special case)

In this sub-case, the transition function $H(h_t) = \phi(h_t)$ is above the 45 degree line for $h_t < \overline{h}$, and for $h_t \geq \overline{h}$, $H(h_t) = \varphi(h_t)$ is linear with a slope greater than 1. Therefore, $H(h_t)$ does not have an interior steady state.

In sum: when $\overline{h} < (\alpha A/R)^{\alpha/(1-\alpha-\beta)}$ holds, it will be possible for the human capital level in the closed economy to take off onto a path of unbounded growth if $\eta \geq 1$, but it will converge to a constant steady state level that is greater than $\overline{h}$ if $\eta < 1$. In any event, the level of human capital will be above the threshold level $\overline{h}$, and hence, the intertemporal externality in human capital formation will be operative in the long run.

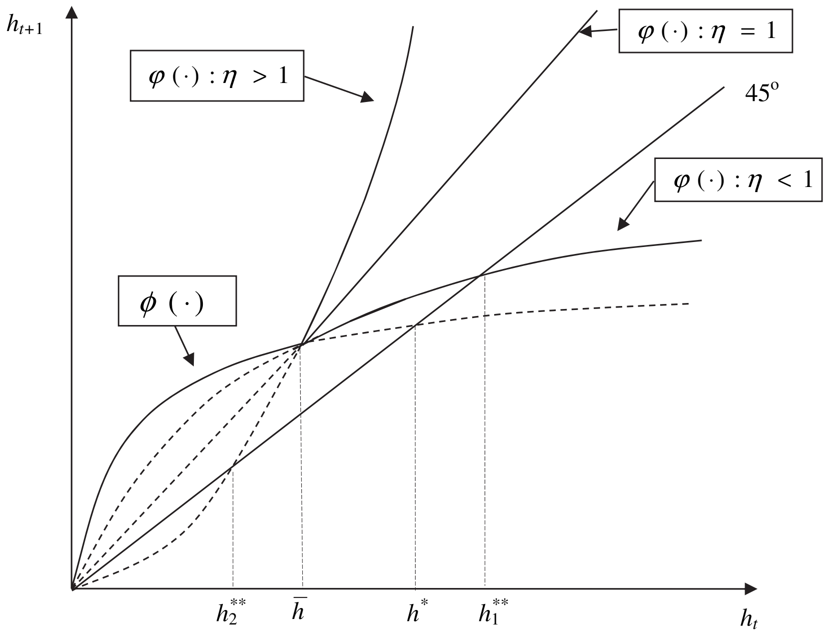

Fig. 3. The phase diagram of the human capital dynamics in the benchmark economy (when $\overline{h} < (\alpha A/R)^{\alpha/(1-\alpha-\beta)}$).

B.2. The growth and welfare effects of migration

Because $\overline{h} < (\alpha A/R)^{\alpha/(1-\alpha-\beta)} = h^*$ holds in the closed economy, Assumption 4 is automatically satisfied under any migration regime with $\overline{\kappa} > 1$, which shifts the phase diagram in Fig. 3 upwards. By assuming, without loss of generality, that the human capital level is above $\overline{h}$, it is rather straightforward to show that

$$\frac{\tilde{g}_t}{g_t} = (1-p)\overline{\kappa}^{\alpha(\beta+\rho)/(1-\alpha)^2}\left(\frac{\tilde{h}_{t-1}}{h_{t-1}}\right)^{(\eta-1)(\beta+\rho)/(1-\alpha)^2}.$$

Recalling the transition functions for human capital with migration and without migration, $\tilde{\varphi}(\cdot)$ and $\varphi(\cdot)$, respectively, it follows that $\tilde{h}_{t-1}/h_{t-1}$ increases without bounds when $\eta = \alpha(1+\rho)+\beta \geq 1$. Therefore, if $\eta \geq 1$, then it follows that $\tilde{g}_t > g_t$ holds for all sufficiently large t. In other words, the growth gain stated in Proposition 1 also carries over to the case $\overline{h} < (\alpha A/R)^{\alpha/(1-\alpha-\beta)} = h^*$.

It can also be shown that in this case

$$\frac{\tilde{c}_{t+1}}{c_{t+1}} = \frac{(1-\alpha\overline{\kappa})\overline{\kappa}^{\alpha/(1-\alpha)}}{(1-\alpha)}\left(\frac{\tilde{h}_t}{h_t}\right)^{(\beta+\rho)/(1-\alpha)}.$$

When $\eta \geq 1$, because $\tilde{h}_t/h_t$ increases without bounds, $\tilde{c}_{t+1} > c_{t+1}$ will hold for sufficient large t. On the other hand, when $\eta < 1$, $\tilde{h}_t$ and h_t converge to the steady states in (17) and (9), respectively, and hence

$$\tilde{c}^*/c^* = \left[(1-\alpha\overline{\kappa})/(1-\alpha)\right]\overline{\kappa}^{\alpha(1+\rho)/(1-\eta)} \equiv \hat{G}(\overline{\kappa}).$$

Noting that $\hat{G}(1) = 1$ and $\hat{G}'(1) > 0$, it follows that $\hat{G}(\overline{\kappa}) > 1$ or, equivalently, $\tilde{c}^* > c^*$, hold for $\overline{\kappa}$ sufficiently close to 1 or, equivalently, p sufficiently close to 0. Therefore, the welfare implications of migration alluded to in the main text carry over to this case as well.

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