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Rethinking the Brain Drain

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Summary. — When productivity is fostered by both the individual's human capital and by the average level of human capital in the economy, individuals underinvest in human capital. A strictly positive probability of migration to a richer country, by raising both the level of human capital formed by optimizing individuals in the home country and the average level of human capital of nonmigrants in the country, can enhance welfare and nudge the economy toward the social optimum. Under a well-controlled restrictive migration policy, the welfare of all workers is higher than in the absence of this policy.

Key words — migration, human capital formation, externalities, social welfare

1. INTRODUCTION

There is a strong consensus that deficiency in human capital is a major reason why poor countries remain poor. Much—though not all—of the human capital in a country is a result of decisions made by individuals. But individual choices seldom add up to the social optimum. In particular, individuals do not consider the positive externalities that human capital confers in production. The result is that they acquire less human capital than is desirable. If individuals could be persuaded to form more human capital, the human capital in an economy could rise to the socially optimal level. What makes an unfortunate state of affairs worse is that whatever quantities of human capital are formed, some—and often more than a mere some—are lost through the migration leakage. It comes as little surprise then that the concern heretofore has been to contain this leakage. In the words of a recent World Development Report: "Can something be done to stop the exodus of trained workers from poorer countries?" (World Bank, 1995, p. 64). This concern follows, and is in congruence with, the large "brain drain" literature (for a systematic review see Bhagwati & Wilson, 1989). The concern is regularly echoed by the informed press. In a May 6, 2000 lead article that addresses the issue of migration to the European Union, *The Economist* magazine states: "[A]ny regime that concentrated on luring the highly skilled would run the risk of robbing poor countries of the people they are least able to do without." In its May 31, 2001 lead article, while advocating the entry of migrants into Europe, *The Economist* hastens to add: "There is a risk, especially when immigration policies target only the highly skilled, that the best talent will be drained from poor countries to rich ones." Similar expressions of alarm are voiced by students of migration. Shkolnikov (1995) writes: "If able younger scientists leave Russia, their older colleagues would have fewer talented people to whom they can pass their knowledge. This could lead to a decline in the quality of research in those scientific disciplines

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where Russia is currently ranked high internationally.'' Although expressed more cautiously, the viewpoint of Carrington and Detragiache (1999) presented in a recent bulletin of the International Monetary Fund is quite similar: ''Another important issue is the extent to which the benefits of education acquired by citizens of developing countries are externalities that individuals cannot be expected to take into account when making their private decisions. If such externalities are substantial, as is emphasized by the 'new growth theory,' then policies to curb the brain drain may be warranted.''

In this paper we turn this concern on its head. We argue that the prospect of migration can well be harnessed to induce individuals to form a socially desirable level of human capital. Our point is that compared to a closed economy, an economy open to migration differs not only in the opportunities that workers face but also in the structure of the incentives they confront; higher prospective returns to human capital in a foreign country impinge on human capital formation decisions at home. We consider a setting in which an individual's productivity is fostered by his own human capital as well as by the economy-wide average level of human capital. We examine the relationship between the actual formation of human capital in an economy and the socially optimal formation of human capital in the economy. We identify conditions under which, from a social point of view, there is too little human capital formation in the economy, and examine the relationship between the actual formation of human capital and the optimal formation of human capital in the presence of a possibility of migration. We identify conditions under which per capita output and the level of welfare of all workers are higher with migration than in its absence, and show that a controlled and restrictive migration policy can enhance welfare and nudge the economy toward the social optimum. We derive this result first, when all workers are alike and are equally capable of responding to the migration prospect, and second, when workers differ both in their skills and in their ability to respond. We conclude that migration is *conducive* to the formation of human capital. Thus, we cast migration as a harbinger of human capital gain, not as the culprit of human capital drain. An interesting implication of our perception of what migration entails is that the gains from migration to the home country accrue neither from migrants' remittances nor from migrants' return home with amplified skills acquired abroad. [1]

The present paper builds on earlier work by Stark, Helmenstein, and Prskawetz (1997, 1998). The main purpose of those papers was to establish a positive causal relationship between the probability of migration by skilled workers from a developing country and human capital formation within the country. The present paper takes several major steps that go beyond the analysis performed in the earlier papers: it defines the socially optimal level of human capital per worker, it measures the difference between that level and the actual level of human capital per worker, it explains the gap between these two levels, it shows that a skillfully managed migration policy serves to eliminate the gap, it identifies a concrete policy tool, it performs explicit welfare analysis, and it presents and investigates the case of a heterogeneous workforce.

2. HUMAN CAPITAL FORMATION IN AN ECONOMY WITHOUT MIGRATION

Consider a closed economy or a small open economy without migration. The economy produces a single commodity. There are N identical workers in the economy. The single production input is labor. The worker's cost function of forming human capital is linear in θ, where θ is the worker's human capital (the sum total of his efficiency units of labor). The economy-wide level of output is N times the per-worker concave production function. This production function is a weighted sum of θ and of $\bar{\theta}$, the economy-wide average level of human capital. The reason for the dependence of the worker's output on $\bar{\theta}$ is the prevalence of externalities that accrue from the average level of human capital. (Externalities in production arise when as a result of individuals acquiring human capital, they not only make themselves more productive but also make each other more productive. Conversely, when individuals fail to form human capital, they not only make themselves less productive but also make each other less productive. A simple way of conditioning a worker's output not only on his own human capital but also on the human capital of others is to have the worker's output depend on the average level of human capital.) Workers supply their human capital inelastically, having acquired it instantly, though not costlessly, at the beginning of their single-period life. Workers borrow the

requisite funds to support the human capital formation at a zero rate of interest.

Since labor is the only production input, the gross earnings per worker are simply equal to the output per worker. The worker seeks to maximize his net earnings, that is, his output minus the cost of forming human capital. Let us refer to the solution to the worker's optimization problem as θ^*. It turns out that θ^* is fully specified by the parameters of the cost function of forming human capital and of the production function. Since there are N identical workers in the economy, the average level of human capital in the economy is also θ^*. Therefore, net earnings per worker are fully specified by the model's parameters. Let us refer to these earnings as $W(\theta^*)$. Since the social returns of human capital are not internalized by the individual worker, θ^* is not the socially optimal level of human capital. Net earnings per worker are maximized when the externalities from the economy-wide average level of human capital are taken into account. The $\bar{\theta}$ that appears in the worker's maximand is substituted by θ in the social planner's maximand. Let us refer to the solution of the social planner's optimization problem as θ^{**}. Two results emerge.

First, $\theta^{**} > \theta^*$. Second, if workers choose to form the socially optimal level of human capital, θ^{**}, the net earnings per worker will become $W(\theta^{**})$. It is easily shown that $W(\theta^{**}) > W(\theta^*)$. Net earnings per worker attained under the social planner's choice of θ are higher than those achieved when workers choose how much human capital to form without taking into consideration the human capital externality. By construction, $W(\theta^{**})$ represents the highest net earnings per worker achievable, given the production technology. Unfortunately, when choosing how much human capital to form, an individual worker will not pay heed to the economy-wide average level of human capital, except as a parameter. In a large economy no individual can affect the economy's average level of human capital. Thus, the prevailing level of human capital will be θ^*.

3. HUMAN CAPITAL FORMATION IN AN ECONOMY WITH MIGRATION

In this section we cast migration policy as a tool to mitigate the inefficiency arising from human capital externalities. Assume that an opportunity to migrate to another, superior technology country, D, presents itself. Assume further that human capital neither depreciates nor appreciates across countries, and that the human capital of individual migrant workers is deciphered in D fully and immediately upon the migrants' arrival. The returns to human capital in D are higher than in the home country, H. A worker's output, and thus his gross earnings, in D are again a concave function of the worker's level of human capital.

Suppose that workers in H face a probability, $p > 0$, of obtaining the gross earnings from an employment in D. With probability $1 - p$ they do not secure such an employment, in which case they work in H for the home country's gross earnings. Again, the worker's decision problem is how much human capital to form. Not surprisingly, the worker's chosen level of human capital, $\tilde{\theta}^*$, depends positively on p. Several results follow.

First, $\tilde{\theta}^* > \theta^*$; in the presence of the possibility of migration, workers choose to form more human capital than in the absence of the possibility of migration. The inducement effect of migration raises the level of human capital of all workers including the workers who stay in H. Thus, the inadequacy of human capital formation due to the externalities is mitigated and consequently welfare can potentially be improved by the possibility of migration. (If the inducement is strong enough, the home country could even be left with more total human capital in the wake of migration. The "brain gain" could then exceed the "brain drain" for the home country's total human capital.)

Second, we are able to perform a complete welfare analysis. To this end, we reason as follows. Since the returns to human capital in D are higher than the returns to human capital in the home country, the net earnings of the workers who migrate to D are higher than the net returns of those who stay behind. (After all, the workers who migrate had incurred exactly the same cost of acquiring human capital as the workers who stay behind, yet the gross earnings of the former are higher than the gross earnings of the latter.) Therefore, the possibility of migration would make every home country worker better off if it makes the nonmigrants better off. To examine whether the possibility of migration made the nonmigrants better off we therefore compare $W(\tilde{\theta}^*)$ and $W(\theta^*)$. Viewing the probability of migration, p, as a policy variable, we can show that the difference between $W(\tilde{\theta}^*)$ and $W(\theta^*)$ attains a unique maximum at a level of p to which we refer as p^*, $0 < p^* < 1$, and that the difference between $W(\tilde{\theta}^*)$ and $W(\theta^*)$ evaluated at p^* is positive.

This result reveals that a carefully designed migration policy can be welfare enhancing and that the welfare gain of the nonmigrants is maximized when the probability of migration is equal to the feasible level p^*. It further follows that when we insert the value of p^* into $\tilde{\theta}^*$, we get that the level of $\tilde{\theta}^*$ is equal to θ^{**}. Therefore, when the probability of migration is p^*, the level of human capital that workers choose to form is exactly the level chosen by the social planner in the absence of migration. Thus, the welfare of the workers who stay behind is inadvertently maximized by the inducement effect of the possibility of migration. It is in this sense that a migration policy can correct for the human capital externality and restore the social optimum.

A skeptic could argue that the optimal probability p^* is a mere theoretical concept; in practice it would be difficult, if not impossible, for the government of the home country to know the exact level of p^*. This may call into question the usefulness of migration as a tool to improve welfare and to correct for the disregard of the human capital externalities. To address this concern we examine the difference between $W(\tilde{\theta}^*)$ and $W(\theta^*)$ as a function of p. We can show that this difference is positive for *any* $0 < p \leqslant p^*$. Thus, as long as the probability of migration is not greater than p^*, the net earnings of a worker who stays in H under migration are *higher* than the net earnings per worker without migration. This suggests the practical use of migration as a welfare-enhancing policy tool even when the government of H does not know the exact level of the optimal probability.

To sum up, our analysis suggests that a controlled and restrictive migration policy can be welfare-enhancing for nonmigrants. In particular, in the presence of a controlled migration policy with the probability of migration set at p^*, the level of human capital that the workers are induced to form turns out to be the socially optimal level of human capital had the workers not migrated.

4. HETEROGENEOUS WORKFORCE, HUMAN CAPITAL FORMATION, AND MIGRATION

The intersection of migration with the presence of externalities could give rise to a concern that those who leave adversely affect the productivity of those who stay behind. If the human capital of the workers who migrate is higher than the human capital of the workers who stay behind, and if a worker's output is an increasing function of the average level of human capital, the nonmigrants will end up worse off; the workers who migrate impose a negative externality on the workers who remain. To address this concern, we examine what might be expected to constitute the worst possible case from the perspective of the low skill workers—the case in which these workers cannot participate in migration at all. We can show that even in such a harsh environment, the human capital formation response of the high skill workers to the migration prospect can still lead to the low skill workers being better off. The essence of the argument is as follows.

Let us relax the assumption that the workforce is homogeneous, and let us suppose that there are two types of workers in H: Low-ability, type-1 workers and high-ability, type-2 workers. Human capital formation is costlier for type-1 workers. Let the cost of forming human capital by a type-1 worker be such that this worker cannot possibly form a level of human capital that is higher than $\underline{\theta}$. The type-2 workers do not face such a constraint and optimally choose to form human capital at the level θ_2^*. If N_1 and N_2 are the numbers of type-1 and type-2 workers, respectively, then (in the absence of migration) the average level of human capital in H is $\bar{\theta} = \frac{N_1\underline{\theta}+N_2\theta_2^*}{N_1+N_2}$.

Let the probability of being selected into employment in D for an H country worker whose human capital is θ be p if $\theta > \underline{\theta}$, and 0 otherwise. The presence of an opportunity to migrate and earn higher wages in D induces the type-2 workers to form more human capital. However, the type-1 workers are immune to this inducement effect because of their inability to form more human capital than the minimal level required for the probable employment in D. Therefore, under the possibility of migration, the levels of human capital formed by type-1 and type-2 workers are, respectively, $\underline{\theta}$ and $\tilde{\theta}_2^*$, where $\tilde{\theta}_2^*$ is an increasing function of p. Hence, the average level of human capital of the workers who remain in H is $\bar{\theta}_m = \frac{N_1\underline{\theta}+(1-p)N_2\tilde{\theta}_2^*}{N_1+(1-p)N_2}$.

We can, first, compare $\bar{\theta}_m$ and $\bar{\theta}$ and derive a reasonable sufficient condition under which the average level of human capital of the nonmigrants in the wake of migration, $\bar{\theta}_m$, is higher than the average level of human capital in the absence of migration, $\bar{\theta}$.

Second, and more important, we can once again perform a complete welfare analysis.

When the migration prospect leads to a higher average human capital, type-1 workers are obviously better off, benefiting from a greater human capital externality. Whether the remaining type-2 workers are also better off under migration is less clear. Yet, when p is small and the "reasonable sufficient condition" yielding $\bar{\theta}_m > \bar{\theta}$ holds, we are able to show that the type-2 workers who remain in H are also better off when the probability of migration is small enough. This result reaffirms the main result of the previous section: a restrictive migration policy can stimulate human capital formation and improve the welfare of all workers. In addition, the possibility of a "brain drain" of high-ability workers from H can confer a positive externality on low-ability workers in H.

5. CONCLUSIONS

When the productivity of an individual in a closed economy or in a small open economy without migration is fostered not only by his own human capital but also by the average level of human capital, the individual who optimally chooses how much to invest in costly human capital formation will, from a social point of view, underinvest. Consequently, social welfare is affected adversely. Somewhat surprisingly, the facility of migration can mitigate this undesirable outcome. In fact, a well-specified migration policy can ameliorate the tendency to underinvest in human capital and permit the formation of a socially desirable level of human capital. The favorable effect of migration and the associated welfare gain apply not only when all individuals can respond to the migration prospect but also when only a subset of individuals are affected. In the latter case, even those who cannot gain from migration by participating in it stand to gain from the response of others.

The propensity to acquire skills is not invariant to the possibility that the skills will be highly rewarded. This consideration appears to have escaped the attention of scholars of migration for many years. The pioneering work of Grubel and Scott (1966) provides a careful account of why a country need not "lose by the emigration of highly skilled individuals." In Grubel's and Scott's words "[E]migration should be welcomed whenever two conditions are met. These are, first, that the emigrant improves his own income and, second, that the migrant's departure does not reduce the income of those remaining behind" (p. 270). That the prospect of migration modifies the human capital formation calculus, thereby entailing a welfare gain for the nonmigrants (rather than being inconsistent with a welfare loss) has neither been mentioned by Grubel and Scott, nor by those who followed in their steps. This paper draws attention to this possible relationship. We have shown that the behavioral response to the prospect of migration nourishes both a "brain drain" and a "brain gain," and that a skillfully executed migration policy can confine and utilize the response to secure a welfare gain for all workers.

6. COMPLEMENTARY REFLECTIONS

Building on the foregoing analysis it could be of interest to assess the sensitivity of our results to alternative specifications, to inquire whether our approach can be extended to incorporate welfare analysis in the destination country, and to consider the policy role that the government of the destination country can play.

In the existing model, human capital is perfectly transferable across economies—moving it does not detract from its productivity (it is perfectly "general"). The existing framework also assumes full employment. Suppose, alternatively, that there are two types of human capital: general, and destination-specific (henceforth "specific"). The latter type is productive abroad but useless at home. The returns to general human capital abroad are considerably higher than the returns to general human capital at home, and the returns to specific human capital are higher still (that is, they are higher than the returns to general human capital abroad). When migration is not a possibility, no worker will acquire specific human capital. Suppose that in such a case every worker optimally acquires $\hat{\theta}$ (of general human capital). When migration is possible and the probability of obtaining gainful employment abroad is $\pi > 0$, and migration into unemployment abroad is not possible, and when the two types of human capital are equally costly to acquire, it should be possible to show that while workers acquire some quantity of specific human capital, because they know that $\pi < 1$ they also acquire a strictly positive quantity of general human capital. (If no general human capital is acquired then, with probability $1 - \pi$, workers will end up unemployed (at home), which would confer an infinite negative utility.) It will be worthwhile to provide conditions under which the level of general human capital

that a worker optimally chooses to form in such an environment, $\hat{\hat{\theta}}$, is *greater* than $\hat{\theta}$, and that welfare, measured by output per worker remaining at home, is also *higher*.

Concerning the general equilibrium analysis suppose, for example, that the destination country's production environment is akin to the home country's production environment. If the level of human capital of the incoming skilled migrant is higher than the average level of human capital in the host country, the effect of human capital externalities in that country will bring about a welfare gain to all the workers there.

The model's insight is not contingent on migration policy formation being exclusively in the hands of the government of the home country, H. Suppose, alternatively, that the enactment of migration policy is in the hands of the government of the destination country, D. Consider a world in which D is keenly interested in raising the level of welfare of the workers of H, can exercise complete discretion as to whether to admit none, few, or many of H's skilled workers, and searches for a migration policy that will raise the welfare of the workers of H by most. Our analysis points to that policy. Moreover, if the welfare gain of the workers of D, to which we referred in the preceding paragraph applies, the choice of p^* by the government of D will not be at the expense of its own workers.

NOTES

1. In the informed press and in public debate, these two counterflows are regularly referred to as the sources of gain that could compensate for the "drain." *The Economist*'s May 6, 2000 lead article states: "Yet even poor countries can benefit when émigrés send home the remittances they earn in the rich world." In an interview held upon assuming the presidency of Harvard University and published in the March 26, 2001 issue of *Newsweek* magazine, Lawrence Summers remarks: "Brain-drain questions are very difficult, but I'm inclined to think that large parts of the answer lie in countries creating economic environments that lead their most able citizens to return home."

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APPENDIX

(a) *Derivation of* θ^*

The purpose of this appendix is to derive the optimal levels of human capital in different settings and to perform comparisons of the measures of welfare that are associated with these levels.

Let the worker's cost function of forming human capital be $C(\theta) = k\theta$ where $k > 0$ is a constant, and let the worker's production function be $f(\theta) = \alpha \ln(\theta + 1) + \eta \ln(\bar{\theta} + 1)$ for $\theta > 0$, where $\alpha > k$ and $\eta > 0$ are coefficients that measure, respectively, the private returns of human capital and the social returns of

human capital. The net earnings per worker function associated with human capital θ is then

$$W(\theta) = \alpha \ln(\theta + 1) + \eta \ln(\bar{\theta} + 1) - k\theta \quad \text{for } \theta > 0.$$

Since

$$\frac{\partial W(\theta)}{\partial \theta} = \frac{\alpha}{\theta + 1} - k$$

$$\left(\text{and } \frac{\partial^2 W(\theta)}{\partial \theta^2} = -\frac{\alpha}{(\theta + 1)^2} < 0 \right),$$

the worker's chosen level of human capital is $\theta^* = \frac{\alpha}{k} - 1$. Thus,

$$W(\theta^*) = (\alpha + \eta) \ln \frac{\alpha}{k} - \alpha + k.$$

(b) *Derivation of* θ^{**}

Taking the externalities from the economy-wide average level of human capital into account, consider the function

$$W(\theta) = \alpha \ln(\theta + 1) + \eta \ln(\theta + 1) - k\theta \quad \text{for } \theta > 0.$$

Since

$$\frac{\partial W(\theta)}{\partial \theta} = \frac{\alpha + \eta}{\theta + 1} - k, \quad \theta^{**} = \frac{\alpha + \eta}{k} - 1.$$

Thus,

$$W(\theta^{**}) = (\alpha + \eta) \ln \frac{\alpha + \eta}{k} - (\alpha + \eta) + k.$$

(c) *A comparison of* θ^{**} *with* θ^*, *and of* $W(\theta^{**})$ *with* $W(\theta^*)$

Since $\eta > 0$, $\theta^{**} > \theta^*$. Since $W(\theta^{**}) - W(\theta^*) = (\alpha + \eta) \ln \frac{\alpha + \eta}{\alpha} - \eta$, and since for any $x > 1$, $x \ln x > x - 1$, it follows, upon substituting $x = \frac{\alpha + \eta}{\alpha} > 1$, that $W(\theta^{**}) - W(\theta^*) > 0$.

(d) *Derivation of* $\tilde{\theta}^*$

Let the returns to human capital in D to an H country worker whose level of human capital is θ be $\beta \ln(\theta + 1) + C$ where $\beta > \alpha + \eta$ and $C \geqslant 0$ are constant and exogenous to the model. The expected net earnings per worker function is

$$\tilde{W}(\theta) = p[\beta \ln(\theta + 1) + C] + (1 - p)[\alpha \ln(\theta + 1) + \eta \ln(\bar{\theta} + 1)] - k\theta.$$

Since

$$\frac{\partial \tilde{W}(\theta)}{\partial \theta} = \frac{p\beta}{\theta + 1} + \frac{(1 - p)\alpha}{\theta + 1} - k = \frac{p(\beta - \alpha) + \alpha}{\theta + 1} - k$$

$$\left(\text{and } \frac{\partial^2 \tilde{W}(\theta)}{\partial \theta^2} = -\frac{p(\beta - \alpha) + \alpha}{(\theta + 1)^2} < 0 \right),$$

the worker's chosen level of human capital is $\tilde{\theta}^* = \frac{p(\beta - \alpha) + \alpha}{k} - 1$. Thus, the level of social welfare, measured by net earnings of the workers who remain in H, is

$$W(\tilde{\theta}^*) = (\alpha + \eta) \ln \frac{p(\beta - \alpha) + \alpha}{k} - [p(\beta - \alpha) + \alpha] + k.$$

(e) *A comparison of* $\tilde{\theta}^*$ *with* θ^*, *and of* $W(\tilde{\theta}^*)$ *with* $W(\theta^*)$

Let

$$G(p) \equiv W(\tilde{\theta}^*) - W(\theta^*) = (\alpha + \eta) \ln \frac{p(\beta - \alpha) + \alpha}{\alpha} - p(\beta - \alpha).$$

Claim. *$G(p)$ has a unique maximum at* $p^* = \frac{\eta}{\beta - \alpha} < 1$, *and* $G(p^*) > 0$.

Proof. Since $G(p)$ is concave, it has a unique maximum. Since

$$\frac{\partial G(p)}{\partial p} = \frac{\alpha + \eta}{p(\beta - \alpha) + \alpha}(\beta - \alpha) - (\beta - \alpha),$$

$$p^* = \frac{\eta}{\beta - \alpha}.$$

Since $\beta > \alpha + \eta$, $p^* < 1$. Inserting p^* into $G(p)$ entails $G(p^*) = (\alpha + \eta) \ln \frac{\eta + \alpha}{\alpha} - \eta$. Upon substituting $x = \frac{\alpha + \eta}{\alpha} > 1$, it follows that $G(p^*) > 0$. □

Notice that

$$\tilde{\theta}^*|_{p = p^*} = \frac{\eta + \alpha}{k} - 1 = \theta^{**}.$$

(f) *A comparison of $W(\tilde{\theta}^*)$ with $W(\theta^*)$ when $0 < p \leqslant p^*$*

Claim. *$G(p) > 0$ for any $0 < p \leqslant p^*$.*

Proof. For any $0 < p \leqslant p^*$, $p(\beta - \alpha) \leqslant \eta$. Thus,

$$G(p) \geqslant [\alpha + p(\beta - \alpha)] \ln \frac{p(\beta - \alpha) + \alpha}{\alpha} - p(\beta - \alpha)$$
$$= \alpha x \ln x - (\alpha x - \alpha) = \alpha[x \ln x - (x - 1)] > 0$$

where $x = \frac{p(\beta-\alpha)+\alpha}{\alpha} > 1$. □

(g) *A comparison of $\bar{\theta}_m$ with $\bar{\theta}$*

A sufficient condition for $\bar{\theta}_m > \bar{\theta}$ to hold is that $(1-p)\tilde{\theta}_2^* > \theta_2^*$, which in turn is true if $(1-p)(\tilde{\theta}_2^* + 1) > \theta_2^* + 1$. But

$$\frac{(1-p)(\tilde{\theta}_2^* + 1)}{\theta_2^* + 1} = 1 + \frac{p[(1-p)(\beta - \alpha) - \alpha]}{\alpha} > 1$$

if $(1-p)(\beta - \alpha) - \alpha > 0$,

or if $0 < p < \dfrac{\beta - 2\alpha}{\beta - \alpha}$.

To ensure that $0 < \frac{\beta-2\alpha}{\beta-\alpha} < 1$, we assume that $\beta > 2\alpha$.

(h) *A comparison of $W(\tilde{\theta}_2^*)$ with $W(\theta_2^*)$*

Let the cost of forming human capital for a type-2 worker be $C(\theta) = k_2\theta$, $0 < k_2 < \alpha$. Then, $\theta_2^* = \frac{\alpha}{k_2} - 1$,

$$W(\theta_2^*) = \alpha \ln \frac{\alpha}{k_2} + \eta \ln(\bar{\theta} + 1) - \alpha + k_2,$$

and

$$W(\tilde{\theta}_2^*) = \alpha \ln \frac{p(\beta - \alpha) + \alpha}{k_2} + \eta \ln(\bar{\theta}_m + 1)$$
$$- [p(\beta - \alpha) + \alpha] + k_2.$$

It follows, then, that

$$G_2(p) \equiv W(\tilde{\theta}_2^*) - W(\theta_2^*)$$
$$= \alpha \ln \frac{p(\beta - \alpha) + \alpha}{\alpha} + \eta \ln \frac{\bar{\theta}_m + 1}{\bar{\theta} + 1} - p(\beta - \alpha).$$

Since

$$\frac{\partial \bar{\theta}_m}{\partial p} = \frac{-N_2\tilde{\theta}_2^* + (1-p)N_2(\beta - \alpha)/k_2}{N_1 + (1-p)N_2}$$
$$+ \frac{[N_1\underline{\theta} + (1-p)N_2\tilde{\theta}_2^*]N_2}{[N_1 + (1-p)N_2]^2},$$

we have that

$$\frac{\partial \bar{\theta}_m}{\partial p}\Big|_{p=0} = \frac{N_2(\beta - 2\alpha)/k_2 + N_2}{N_1 + N_2}$$
$$+ \frac{(N_1\underline{\theta} + N_2\theta_2^*)N_2}{(N_1 + N_2)^2} > 0,$$

where the inequality follows from the assumption that $\beta > 2\alpha$. Drawing on this inequality, we differentiate $G_2(p)$ with respect to p and evaluate the result at $p = 0$ to obtain that

$$\left.\frac{\partial G_2(p)}{\partial p}\right|_{p=0} > 0.$$

By continuity, $G_2(p) > 0$ holds for p in a small positive neighborhood of zero.