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Dynasties and Destiny: On the Roles of Altruism and Impatience in the Evolution of Consumption and Bequests*

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We study the joint role of altruism and impatience, and the impact of evolution in the formation of long-term time preferences and in the determination of optimal consumption and optimal bequests. We show how the consumption paths of dynasties relate to altruism and to impatience, and we reason that long-lived dynasties will be characterized by a higher degree of altruism and a lower degree of impatience than short-lived dynasties.

INTRODUCTION

Consider an individual who inherits a forest. Year by year, under the auspices of Mother Nature, the forest grows. The individual has to choose how much wood to consume during his lifetime and how much of the forest to bequeath to his children. What governs the choice? Given the parameters that impinge on the individual's choice, what determines the intra- and intergenerational consumption paths? In the long run, when evolutionary pressures manifest themselves, which parameter configuration is likely to prevail?

Several studies have addressed the topic of (resource) allocation across generations and the appropriate generational weights associated with such an allocation. Arrow (1973) and Dasgupta (1974) use a 'private' utility function of parents and their offspring to form the following welfare function of the t-period generation:

$$W_t = \sum_{i=0}^{m} \beta_i U(c_{t+i}),$$

where i denotes generation, $U(c_{t+i})$ denotes the utility of the $(t+i)$th generation from consumption, m is the number of the generations that the t-period generation accounts for (in Dasgupta's model, $m = 1$), and $0 < \beta_i \leqslant 1$ is the generational discount rate such that $\beta_i \geqslant \beta_{i+1}$. What constitutes β_i is not specified. Barro (1974) models the altruistic concern of individuals for the welfare of their offspring in an overlapping-generations economy where the planning horizon extends to infinity and population size is fixed. The utility function of a member of the ith generation is given by $U_i = U((c_i^y, c_i^o), U_{i+1}^*)$ where c_i^o is consumption of the old in generation i, c_i^y is consumption of the young (the offspring) in generation i, and U_{i+1}^* is the optimal utility of the offspring in the next generation. What governs the explicit form of $U(\cdot, U_{i+1}^*)$ is not expounded. Barro and Becker (1989) use a linear multi-generational utility of altruistic individuals with several offspring where the planning

* This paper is dedicated to Meir Liebergall, M.D., and Ram Mosheiff, M.D., shapers of destinies and practitioners of altruism.

horizon extends to infinity. Utility is defined as $U_i = V(c_i) + a(n_i)n_i\, U_{i+1}$, where i is the generational index, c_i is individual i's own consumption, $V(c_i)$ is the individual's direct utility from his consumption, n_i is the number of children the individual has, $a(n_i)$ is the degree of altruism of the individual towards each of his children, and U_{i+1} is the utility attained by each child. While this formulation sheds more light on what underlies intergenerational weighting and discounting, what forms the $a(n_i)$ weight is not characterized.

In this paper we use a piecewise-continuous utility function that is akin to the time-discrete utility functions of Arrow, and of Barro and Becker. However, our function differs from theirs in that it is defined over a composite separable measure of intergenerational altruism and intragenerational impatience. Unlike Barro, and Barro and Becker, we define a preference order which is not confined to a planning horizon that extends to infinity. The planning horizon itself is a preference parameter that impinges on consumption and on bequests. In addition, we closely analyse the roles of altruism and impatience in an evolutionary environment. In particular, we model the long-term time preferences and the choices of individuals who are both impatient (Koopmans 1960) and altruistic (Barro and Becker 1989) towards their children.

We inquire how the individual's preferences that incorporate intertemporal altruism, intratemporal impatience, and intertemporal farsightedness determine the individual's consumption and bequest. We construct a model that enables us to derive the individual's optimal level of consumption, optimal consumption path, and optimal bequest. We show how these magnitudes relate to a composite measure of altruism and impatience, and why altruism and patience fulfil similar intertemporal roles. We present conditions under which, in the life of a dynasty, bequests are higher than inheritances; altruism penalizes the consumption of early generations but enhances the consumption of late generations; and, holding age constant, consumption rises in the generational order. In particular, we show that although in the short run members of a dynasty emanating from, and replicating the preferences of, an individual who is more altruistic face a lower level of consumption than members of a dynasty emanating from a less altruistic individual, in the long run members of the former dynasty inherit, consume, *and* bequeath more than members of the latter dynasty. This finding prompts us to conjecture that since higher altruism confers an evolutionary edge in the long run, an altruistic inclination can become the prevailing trait in a population. We derive a similar conjecture with regard to patience.

Three studies—Weil (1987), Vidal (1996), and Dutta and Michel (1998)—address topics that bear closely on the issues we investigate, yet obtain results that differ somewhat from ours. It is useful to highlight briefly the similarities and denote the differences.

Weil (1987) studies consumption dynamics in an economy characterized by overlapping generations with a bequest motive (parents care about their children's utility) and investigates the applicability of Barro's (1974) debt neutrality proposition. Building on Weil's model, Vidal (1996) studies the long-run distribution of dynasties (social classes) in an economy characterized by heterogeneity across dynasties in the altruism parameter. In these models, the optimal bequest appears to be negatively related to a measure of patience. This result seems to contrast with our finding, and for that matter with Becker's (1980) finding long before us. The reason for the difference lies in the fact that our equivalent of the intergenerational discount factor of Weil and Vidal is a composite measure of altruism and impatience (and implicitly also of fertility). It can be demonstrated that when such a measure is incorporated in the models of Weil and Vidal, bequests are positively related to patience.[1] (In the hybrid model we also show that Weil's result that the long-run interest rate does not depend on the intertemporal discount factor but only on the intergenerational discount factor warrants a modification.)

Our model is free from the requirement of an operative bequest constraint discussed in Weil, Vidal, Becker and many others, because ours is a dynamically efficient economy wherein the shadow price of capital is equal to the rate of return to capital (given by the forest growth rate), and is independent of the value of the altruism parameter. An operative bequest constraint needs to be incorporated when the economy is dynamically inefficient. Note that if each individual can survive on wage earnings alone, an optimizing individual whose intergenerational discount factor is low could well prefer to leave a negative bequest, should this be feasible.

Becker (1980) and Vidal (1996) show that in the long run all capital ends up in the hands of the class of dynasties whose intergenerational discount factor is the highest. All other dynasties live on wages alone, leaving and receiving no bequests. In contrast, because our model assumes that owning capital is necessary for survival, we find that in the long run most of the capital ends up in the hands of the most altruistic dynasties, with strictly positive quantities of capital distributed (unequally) across the other dynasties.

Dutta and Michel (1998) present a model wherein heterogeneity in preferences arises intertemporally within dynasties but not across dynasties. Dutta and Michel's concept of heterogeneity differs from our concept of heterogeneity (or, for that matter, from Vidal's). In Dutta and Michel's population of dynasties, the intergenerational discount in each dynasty—a measure of altruism—can take one of two arbitrary values: 1 and 0. In the long run the proportion of altruists in the population is constant and the wealth distribution is stationary. The stationary equilibrium wealth distribution can be degenerate, implying perfect equality. In our setting, however, there is a growing dispersion of wealth.

I. Preliminaries

We consider a forward-looking individual. The individual has a long-term utility function defined over a multi-generational horizon. N is the number of generations ahead that the individual considers. It measures the individual's ability or proclivity to imagine and relate to the future. The length of the life-span of a generation (the generation's lifetime) is normalized as 1.

To simplify, we assume that every individual has one child and that every child has one parent. Let α denote the intergenerational weight the individual assigns to the utility of his child. It is a measure of the individual's altruism towards his descendant. We discuss the upper bound of α in Section II.

Let U be the individual's long-term utility function. It is the sum of the generational utility functions, $W_n, n = 0, 1, 2, \ldots, N$:

$$(1) \qquad U = \sum_{n=0}^{N} W_n.$$

W_0 is the utility the individual derives from consumption throughout his own lifetime, W_1 is the utility of the individual from the consumption of his child, and so on.

The utility of the individual from the consumption of his nth removed descendant, W_n, is defined as follows:

$$(2) \qquad W_n = \alpha^n \int_{t=n}^{n+1} e^{-\delta t}\, u(c_t)\, dt, \qquad n = 0, 1, \ldots, N,$$

where $u(c_t)$ is an intragenerational concave utility function, defined over timely consumption, c_t, and t stands for time. $\delta > 0$ is the individual's degree of impatience. It captures the individual's pure subjective discounting of future consumption. (If $\delta = 0$, an optimizing individual may elect to postpone consumption in a manner that can endanger his own life and thereby the very continuity of his dynasty.) α^n measures the weight the individual assigns to the utility of his nth descendant. The individual is of the opinion that the utility weights assigned by his descendants and their degree of impatience mimic his.

The equivalent measure in our model to the time-discrete generational discount factor of Arrow (β_i) and of Barro and Becker ($a(n_i)$) is $\alpha e^{-\delta}$, a composite measure of intergenerational altruism and intragenerational impatience.[2] Like Arrow's model, but unlike Barro and Becker's, our model is not confined to a farsightedness measure that tends to infinity. (Note that the order of preference preserved by U is connected, reflexive, transitive, continuous, and strongly monotonic if and only if the order preserved by $u(c_t)$ is connected, reflexive, transitive, continuous, and strongly monotonic for all c_t.)

Let K_t measure the consumption source at time t. The individual's starting endowment (the forest bequeathed to him), K_0, is given. Let the constant, exogenous, and given rate of return (the forest growth rate) be r. The dynamics of K is:

$$(3) \qquad dK_t/dt = rK_t - c_t, \qquad 0 \leqslant t \leqslant N + 1.$$

Let I_n denote the present value of the consumption in generation n,

$$(4) \qquad I_n = \int_{t=n}^{n+1} e^{-rt} c_t dt, \qquad n = 0, 1, \ldots, N.$$

Since the sum of the present values of all generational consumptions cannot exceed the individual's starting endowment, we have

$$(5) \qquad K_0 \geqslant \sum_{n=0}^{N} I_n.$$

Denote by H_1 the present value of the consumption source the individual bequeaths to his descendants. Then

$$(6) \qquad H_1 = K_0 - I_0 = e^{-r} K_1,$$

where K_1 is the value of the consumption source at the time of the individual's passing.

II. The Piecewise-Continuous Maximization Problem

The individual wishes to maximize his long-term utility. His decision variables are c_t, for $0 \leqslant t \leqslant 1$, and H_1, the present value of the bequest he leaves behind. In order to calculate the optimal value of H_1, the individual maximizes his long-term utility function, over c_t, for the time horizon $0 \leqslant t \leqslant N+1$. The target functional, J, is

$$J = \sum_{n=0}^{N} \alpha^n \int_{t=n}^{n+1} e^{-\delta t} u(c_t)\, dt. \tag{7}$$

The target functional is maximized over c_t for all values of t under consideration, subject to the state equation (3) and the starting endowment K_0.

In the Appendix, the maximization problem is solved in two steps. In the first step the optimal allocation of c_t within each of the $N+1$ generations is calculated, assuming that the values of I_n, $n = 0, 1, \ldots, N$, are given, and subject to (4). In the second step the optimal values of I_n are calculated, given the consumption allocations obtained in the first optimization step.

The optimal values of c_t (given by (A8) in the Appendix) and I_n (given by (A12)) are

$$c_t = \frac{I_n \delta\, e^{(r-\delta)t}}{1 - e^{-\delta}}, \qquad n = 0, 1, \ldots, N, \tag{8}$$

and

$$I_n = \frac{K_0 \alpha^n e^{-\delta n}}{\sum_{n=0}^{N} \alpha^n e^{-\delta n}}, \tag{9}$$

and the present value of the individual's bequest, H_1 (given by (A13)) is

$$H_1 = K_0 - I_0 = K_0 \left(1 - \frac{1}{\sum_{n=0}^{N} \alpha^n e^{-\delta n}} \right). \tag{10}$$

Our framework implies an upper limit of the altruism coefficient. It is clear from (10) that the sum

$$\sum_{n=0}^{N} \alpha^n e^{-\delta n}$$

must be finite; otherwise the present value of what the individual bequeaths, H_1, is equal to his starting endowment, K_0, rendering the present value of his consumption $I_0 = 0$, thus jeopardizing his life. Since we allow $N \rightarrow \infty$, $\alpha e^{-\delta}$ must be strictly smaller than 1 to ensure a finite sum, that is, α must be strictly smaller than e^{δ}. (Since δ is strictly positive, $e^{\delta} > 1$.) If we restrict the discussion to values of α that do not exceed 1, the sum in question will be finite for any $\delta > 0$.[3]

What is the value of the consumption source, K_1, at time $t = 1$ when the bequest is made? From (6) and (10) it follows that the (time-consistent) value is

$$(11) \qquad K_1 = \mathrm{e}^r H_1 = \mathrm{e}^r K_0 \left(1 - \frac{1}{\sum_{n=0}^{N} \alpha^n \mathrm{e}^{-\delta n}} \right).$$

The optimal value of K_1 depends on the altruism coefficient, α, the planning horizon, N, the degree of impatience, δ, and the exogenous rate of return, r. In particular, K_1 is positively related to the altruism coefficient:

$$(12) \qquad \frac{\partial K_1}{\partial \alpha} = \mathrm{e}^r K_0 \left[\left(\sum_{n=0}^{N} \alpha^n \mathrm{e}^{-\delta n} \right)^{-2} \left(\sum_{n=0}^{N} n\alpha^{n-1} \mathrm{e}^{-\delta n} \right) \right] > 0.$$

The higher the weight the individual attaches to the wellbeing of his child, the larger the bequest to the child. We can also sign the other three dependencies.

First, K_1 is positively related to the planning horizon:

$$(13) \qquad \frac{\Delta K_1}{\Delta N} = \mathrm{e}^r K_0 \left[\left(\sum_{n=0}^{N-1} \alpha^n \mathrm{e}^{-\delta n} \right)^{-1} - \left(\sum_{n=0}^{N} \alpha^n \mathrm{e}^{-\delta n} \right)^{-1} \right] > 0.$$

The longer the planning horizon, the larger the bequest.

Second, K_1 is negatively related to the individual's degree of impatience:

$$(14) \qquad \frac{\partial K_1}{\partial \delta} = -\frac{\mathrm{e}^r K_0 \left[\sum_{n=0}^{N} (\alpha^n \mathrm{e}^{-\delta n})\, n \right]}{\left[\sum_{n=0}^{N} (\alpha^n \mathrm{e}^{-\delta n}) \right]^2} < 0.$$

The role of impatience in intergenerational transfers is similar to the role of impatience in intragenerational transfers: higher impatience entails leaving less to the future.

Finally, K_1 is positively related to the rate of return:

$$(15) \qquad \partial K_1 / \partial r = K_1 > 0.$$

A higher rate of return confers a higher yield which facilitates a larger intergenerational transfer.

III. Implications

What can we learn from our modelling framework about the relationship between the inheritance received by the individual, K_0, and the bequest made by him, K_1? Using (11), we obtain

$$(16) \qquad \frac{\partial K_1}{\partial K_0} = e^r \left(1 - \frac{1}{\sum_{n=0}^{N} \alpha^n e^{-\delta n}} \right) = \frac{K_1}{K_0} > 0.$$

Thus, an optimizing individual who receives a larger K_0 ends up leaving a larger K_1. What might have been attributed to an abstract notion of fairness appears to arise from hard-nosed optimization. Moreover, we can determine under which configuration of parameters K_1 will be *larger* than K_0. For such a relationship to hold, the right-hand side of (16) has to be larger than 1, that is,

$$(17) \qquad \sum_{n=0}^{N} \alpha^n e^{-\delta n} > (1 - e^{-r})^{-1}.$$

This relationship is more likely to hold the larger is α, the larger is N, the smaller is δ, and the larger is r.

By evaluating (11) at $N \to \infty$, we obtain

$$(18) \qquad \frac{K_1}{K_0} = \alpha e^{r - \delta}.$$

We have that α must be equal to $e^{\delta - r}$ for K_1 to be equal to K_0. Whenever $\delta = r$, α must be equal to 1 in order for K_1 to be equal to K_0. However, if $\alpha < 1$, 'intergenerational equality' is obtained if and only if the rate of return exceeds the degree of impatience. This result can be reasoned as follows. Intergenerational mental discounting is a composite measure of impatience and altruism. Intergenerational equality is obtained whenever mental discounting, given by $\alpha e^{-\delta}$, is equal to the exogenous discounting given by e^{-r}. From (18) we also see that since both a large α and a low δ operate in the direction of raising K_1/K_0, $K_1 > K_0$ can be maintained for a lower α if δ is lower or, for a higher δ (but not as high as r) if α is higher.

We next ask: does an individual who is more altruistic towards his descendants consume less than one who is less altruistic (so as to facilitate a larger bequest)? And yet, in the long run, do those dynasties characterized by a higher altruism coefficient consume more because, generation after generation, the inheritances received are larger?

To answer the first question, we rewrite c_t (substituting the optimal value of I_0 from (9) into the consumption function (8)) to obtain

$$(19) \qquad c_t = \frac{K_0 \delta \alpha^n e^{-\delta n} e^{(r-\delta)t}}{(1 - e^{-\delta}) \sum_{n=0}^{N} \alpha^n e^{-\delta n}}, \qquad 0 \leqslant t \leqslant 1.$$

Differentiating c_t with respect to the altruism coefficient and evaluating the derivative at $n = 0$ gives

$$(20) \qquad \left.\frac{\partial c_t}{\partial \alpha}\right|_{n=0} = -\frac{K_0 \delta e^{(r-\delta)t} \sum_{n=0}^{N} n\alpha^{n-1} e^{-\delta n}}{(1-e^{-\delta})\left(\sum_{n=0}^{N} \alpha^n e^{-\delta n}\right)^2} < 0, \qquad 0 \leqslant t \leqslant 1.$$

Consequently, the answer to our first question is that a more altruistic (first-generation) individual does indeed consume less.

As to our second question, note that the individual's descendant, $n = 1$, benefits from his parent's higher altruism since the descendant's consumption positively relates to K_1,[4] and K_1, in turn, positively relates to the individual's altruism (recalling (12)). However, other things remaining the same, the descendant's consumption is penalized by his higher altruism. The bequest he leaves, K_2, gains from being positively related to K_1, as well as from the descendant's heightened altruism. The consumption of the next descendant down thus gains from the higher altruism of the preceding *two* generations, while it is penalized by the second descendant's own increased altruism. Suppose that this reasoning is iterated period after period well into the future. The consumption of the nth descendant gains from the higher altruism of all the preceding $n - 1$ generations, and is penalized by the nth descendant's own increased altruism. We can now visualize the intragenerational consumption profile in a dynasty emanating from an individual whose α is high, as opposed to the intragenerational consumption profile in a dynasty emanating from an individual whose α is relatively low. Starting with the 'founding' individuals, the consumption profile of the first dynasty will be steeper and will cut from below the consumption profile of the second dynasty. Moreover, several generations later, a member of the first dynasty not only will consume more during his lifetime than a member of the second dynasty, but also will bequeath more.

To trace the dynamics of consumption as $N \rightarrow \infty$ we rewrite c_t once more (drawing on Appendix equations (A5) and (A7), and on (9)) to obtain

$$(21) \qquad c_t = \frac{K_0 \alpha^n \delta(1-\alpha e^{-\delta})\, e^{(r-\delta)t}}{(1-e^{-\delta})}.$$

We see that the consumption of an individual depends upon the generation he belongs to (n) and upon his age ($t - n$). (Note that $e^{(r-\delta)t} = e^{(r-\delta)n}\, e^{(r-\delta)(t-n)}$.) In particular, the consumption ratio for two individuals of the same age across any two successive generations is given by $\alpha e^{r-\delta}$. Hence, if $\alpha e^{r-\delta} > 1$, consumption for any given age rises in the generational order. The dynamics of the intragenerational consumption is determined by the relationship between the impatience coefficient and the rate of return: whenever $\delta < r$, consumption rises during an individual's lifetime; whenever $\delta = r$, the intragenerational consumption is constant in age; and whenever $\delta > r$, consumption is negatively related to age.

To study the evolution of the consumption of dynasties with different degrees of altruism as $N \longrightarrow \infty$, we differentiate c_t in (21) with respect to α to obtain

$$\frac{\partial c_t}{\partial \alpha} = \frac{K_0 \alpha^{n-1} \delta n (1 - \alpha e^{-\delta} - \alpha e^{-\delta}/n)}{(1 - e^{-\delta})}. \tag{22}$$

The right-hand side of (22) is negative for $n = 0$ but becomes positive for $n > \alpha e^{-\delta}/(1 - \alpha e^{-\delta})$; stronger altruism penalizes consumption early in the life of a dynasty but enhances it from some future time.

Let $v(\alpha^1, \alpha^2)$ denote the index of the generation at which the consumption of a dynasty with a higher altruism coefficient, α^1, first surpasses the consumption of a dynasty with a lower altruism coefficient, α^2; v is the smallest natural number for which $c_v(\alpha^1)$ exceeds $c_v(\alpha^2)$. Using (21), we obtain

$$v(\alpha^1, \alpha^2) \geqslant \frac{\ln[(1 - \alpha^2 e^{-\delta})/(1 - \alpha^1 e^{-\delta})]}{\ln(\alpha^1/\alpha^2)}, \qquad v = 1, 2, 3, \ldots . \tag{23}$$

Note that $v(\alpha^1, \alpha^2)$ exists, it is positive, and it is finite for all $\alpha^1 \leqslant 1$. Figure 1 compares the evolution of the intergenerational consumptions of the two

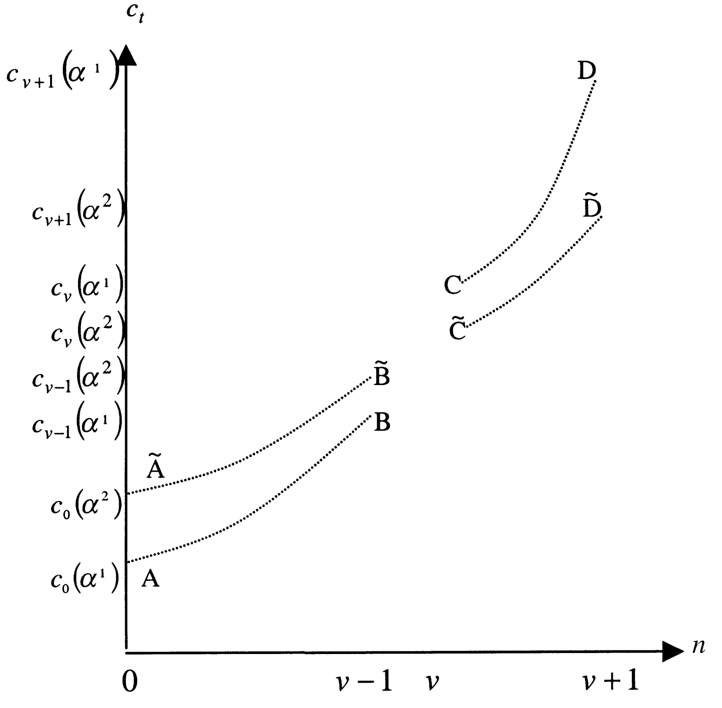

FIGURE 1. The evolution of intergenerational consumption of two dynasties with $\alpha^i > e^{\delta - r}$, $i = 1,2$ and $\alpha^1 > \alpha^2$.

Points A, B, C, and D portray the consumption of the dynasty with the higher altruism coefficient, at the beginning of generations 0, $(v-1)$, v, and $(v+1)$, respectively. Points Ã, B̃, C̃, and D̃ portray consumption at the beginning of the same generations of the second dynasty. The lines connecting the consumption at the beginning of each generation's lifetime, for either dynasty, are imaginary—they do not depict intragenerational consumption.

dynasties. The figure is drawn for $\alpha^i e^{r-\delta} > 1$ and $\alpha^1 > \alpha^2$. The first generation of the dynasty with the higher altruism coefficient consumes less than the first generation of the dynasty with the low altruism coefficient. However, the growth of consumption across generations, holding age constant, which is given by $\alpha e^{r-\delta}$, is positively related to the degree of altruism. Therefore, after the first $v-1$ generations, the consumption of a dynasty characterized by high altruism exceeds the consumption of a dynasty characterized by low altruism.

To study the evolution of the consumption of dynasties with different degrees of impatience as $N \longrightarrow \infty$, we mimic the steps we have taken to study the role of different degrees of altruism.

By differentiating c_t in (21) with respect to δ, we obtain

$$\frac{\partial c_t}{\partial \delta} = \frac{K_0 \alpha^n e^{(r-\delta)t}}{(1-e^{-\delta})^2}[(1-\alpha e^{-\delta})(1-e^{-\delta}-\delta e^{-\delta}) + \alpha\delta e^{-\delta}(1-e^{-\delta}) - t\delta(1-\alpha e^{-\delta})(1-e^{-\delta})]. \tag{24}$$

Since $e^{\delta} > 1+\delta$ for all positive values of δ, $(1-e^{-\delta}-\delta e^{-\delta}) > 0$. Thus, $\partial c_t/\partial\delta$ is positive at $t=0$, the beginning of the individual's life, but for $t \geqslant t_\delta$, where t_δ satisfies

$$t_\delta > \frac{[(1-\alpha e^{-\delta})(1-e^{\delta}-\delta e^{-\delta}) + \alpha\delta e^{-\delta}(1-e^{-\delta})]}{[(1-\alpha e^{-\delta})(1-e^{-\delta})]}, \tag{25}$$

$\partial c_t/\partial\delta$ turns negative. Note that t_δ exists and is finite. Impatience has both intragenerational and intergenerational effects. Greater impatience gives added weight to the individual's immediate consumption which can be highly beneficial to the individual's survival. Greater patience relegates more consumption to the future at the expense of earlier consumption. From some future point in time (which may or may not fall within the individual's own lifetime), greater patience enhances the consumption of the individual's dynasty. Note that if $\alpha = 0$, all the benefits from greater patience are reaped by the individual during his own lifetime. (Future generations cannot possibly enjoy the fruits of the individual's greater patience if the individual is not altruistic.)

To track more closely the intergenerational role of patience, we compare the consumption of a dynasty with a lower impatience rate, δ^1, with the consumption of a dynasty with a higher impatience rate, δ^2. Let $\eta = 0, 1, 2, \ldots$ denote the index of the generation at which the consumption of the dynasty with δ^1 first surpasses the consumption of the dynasty with δ^2. Using (21), we get

$$e^{-(\delta^2-\delta^1)\eta} \leqslant \frac{\delta^1(1-\alpha e^{-\delta^1})(1-e^{-\delta^2})}{\delta^2(1-\alpha e^{-\delta^2})(1-e^{-\delta^1})} < e^{-(\delta^2-\delta^1)(\eta-1)}. \tag{26}$$

Note that $\eta = \eta(\delta^1, \delta^2)$ exists and is finite for all feasible values of α and δ.

We conclude that after an initial, finite period during which consumption is taxed by greater patience, dynastic consumption benefits from enhanced patience. In addition, the impatience rate that maximizes dynastic consumption tends to zero.

Pulling together the results from the analyses of the effects of a stronger altruism and a greater patience we see that in the intergenerational context, the altruism and patience of the founder of a dynasty and the replication of that altruism and patience by his descendants pay off. Not only are bequests higher but in successive generations and thereafter, all dynasty members enjoy higher levels of consumption than members of a dynasty whose founder was less altruistic and patient. From some generation on, consuming more is congruent with bequeathing (hence inheriting) more, not at the expense of bequeathing (hence inheriting) less. In addition, the value of altruism that maximizes the consumption of a dynasty differs from the value of altruism that maximizes the consumption of the individual. Moreover, the value of impatience that maximizes the consumption of a dynasty differs from the value of impatience that maximizes the consumption of the individual.

IV. Conclusions

Economists, biologists, philosophers, and others have long pondered whether altruism is detrimental to survival or whether it confers a survival advantage. If consumption positively affects the probability of survival, a dynasty whose members consume more will have an edge over a dynasty whose members consume less, such that in the long run the first dynasty's chances of survival are higher. Our analysis points to the positive role of altruism in this regard. Indeed, the longer the long run, the more pronounced the edge (the wider is the inter-dynasty consumption wedge). It follows then that in a short-lived society the share of low-empathy individuals will be higher than in a long-lived society: the share of altruists in a society correlates positively with its age. Note, however, that if the likelihood of awfully bad states of nature occurring *in the short run* (that is, before the altruism-induced and the patience-induced advantages kick in) is high, which hitherto we have implicitly assumed not to be the case, our conclusions will need to be revised.

Our analysis also reveals an interesting relationship between altruism and impatience. In the evolution of consumption and bequests, altruism and patience play similar roles, and more of one can substitute for less of the other. In addition, altruism enhances the long-run benefits of patience. Since high altruism and low impatience confer the highest advantage in the long term (measured in terms of the level of consumption), such dynasties will have the strongest edge. Thus, in long-lived dynasties, altruism and patience will co-exist; evidence of one could suggest the presence of the other.

In the survival game, long-lasting genes appear to play an important role. A dynasty whose members 'carry' the altruism and patience traits that optimize the dynasty's consumption rather than their own consumption has a better chance of withstanding the process of natural selection. One reason why we are (somewhat) altruistic and (somewhat) patient is that we are descendants of dynasties that survived to the present. Those who are wholly non-altruistic and wholly impatient belong to dynasties that no longer exist.

APPENDIX

The Hamiltonian functions of the first maximization step, H_t, are

$$(A1)\qquad H_t = \begin{cases} e^{-\delta t}u(c_t) + q_t^1(rK_t - c_t) & 0 \leqslant t \leqslant 1 \\ \alpha e^{-\delta t}u(c_t) + q_t^1(rK_t - c_t) & 1 \leqslant t \leqslant 2 \\ \vdots & \vdots \\ \alpha^N e^{-\delta t}u(c_t) + q_t^1(rK_t - c_t) & N \leqslant t \leqslant N+1, \end{cases}$$

where q_t^1 denotes the adjoint variable. Maximizing H_t with respect to c_t gives

$$(A2)\qquad q_t^1 = \begin{cases} e^{-\delta t}u'(c_t) & 0 \leqslant t \leqslant 1 \\ \alpha e^{-\delta t}u'(c_t) & 1 \leqslant t \leqslant 2 \\ \vdots & \vdots \\ \alpha^N e^{-\delta t}u'(c_t) & N \leqslant t \leqslant N+1. \end{cases}$$

The Euler equations, $-\partial H_t/\partial K_t = dq_t^1/dt$, regulate the dynamics of consumption *within* each generation:

$$(A3)\qquad \frac{(\delta - r)u'(c_t)}{u''(c_t)} = \frac{dc_t}{dt}, \qquad n \leqslant t \leqslant n+1.$$

Note that c_t, dc_t/dt, and $u'(c_t)$ are piecewise-continuous functions; they are continuous over the time horizon stretching from 0 to $N+1$ except for $t = 1, 2, \ldots, N$. At the points of discontinuity, left and right derivatives, dc_t/dt as well as $u'(c_t)$, differ. In addition, c_t admits two (closure) values, one referring to consumption of the old generation ($n = t-1$) and the other referring to consumption of the new generation ($n = t$).

Suppose that $u(c_t) = \ln(c_t)$. Equation (A3) then becomes

$$(A4)\qquad dc_t/dt = (r - \delta)c_t, \qquad n \leqslant t \leqslant n+1.$$

The consumption planned by the individual for generation n, for every n under consideration, is then

$$(A5)\qquad c_t = c_n e^{(r-\delta)t}, \qquad n \leqslant t \leqslant n+1,\ n = 0, 1, \ldots, \mathrm{N},$$

where c_n is consumption at the beginning of the lifetime of generation n. By substituting (A5) into (4) we find that c_n satisfies

$$(A6)\qquad I_n = \int_{t=n}^{n+1} e^{-rt} c_n e^{(r-\delta)t}\, dt.$$

The explicit value of c_n is thus

$$(A7)\qquad c_n = \frac{I_n \delta}{e^{-\delta n}(1 - e^{-\delta})}.$$

Substituting (A7) into (A5), we obtain the explicit value of c_t:

$$(A8)\qquad c_t = I_n \left[\frac{\delta}{e^{-\delta n}(1 - e^{-\delta})}\right] e^{(r-\delta)t}, \qquad n = 0, 1, \ldots, N.$$

Denote by g_n the bracketed term in (A8). From (2) and (A8), and recalling that $u(c_t) = \ln(c_t)$, we can express the optimal W_n in terms of I_n and g_n:

$$(A9)\qquad W_n = \alpha^n \int_{t=n}^{n+1} e^{-\delta t}[\ln(I_n) + \ln(g_n) + (r - \delta)t]\, dt, \qquad n = 0, 1, \ldots, N.$$

The second-order condition (the Legenre condition) is satisfied whenever the second derivative of the Hamiltonian with respect to c_t is negative, for all values of t under consideration. It is easily verified that for our logarithmic utility function, the condition is indeed satisfied.

We now turn to the second step of the maximization problem—calculating the optimal values of I_n for all n under consideration, given the optimal W_n in (A9), which we express as $W_n(I_n)$. The Lagrangian, L, of this time-discrete maximization problem is

$$\text{(A10)} \quad L = \sum_{n=0}^{N} W_n(I_n) + \mu\left(K_0 - \sum_{n=0}^{N} I_n\right),$$

where μ is the Lagrangian multiplier. The first-order conditions are

$$\text{(A11a)} \quad \mu = \partial W_n(I_n)/\partial I_n, \qquad n = 0, 1, \ldots, N,$$

and

$$\text{(A11b)} \quad K_0 \geqslant \sum_{n=0}^{N} I_n.$$

The $(N+1)$ equations in (A11a) can be summarized as follows:

$$\frac{1}{I_0} = \frac{\alpha \mathrm{e}^{-\delta}}{I_1} = \cdots = \frac{\alpha^N \mathrm{e}^{-\delta N}}{I_N}.$$

The second-order condition is satisfied whenever the second derivative of L with respect to I_n, for each n, is negative. It is easily verified that in our case the condition is indeed satisfied.

Using (A11a) and (A11b) we can express I_n in terms of K_0 as follows:

$$\text{(A12)} \quad I_n = \frac{K_0 \alpha^n \mathrm{e}^{-\delta n}}{\sum_{n=0}^{N} \alpha^n \mathrm{e}^{-\delta n}}.$$

From (6) and (A12), the present value of the individual's bequest, H_1, is

$$\text{(A13)} \quad H_1 = K_0 - I_0 = K_0\left(1 - \frac{1}{\sum_{n=0}^{N} \alpha^n \mathrm{e}^{-\delta n}}\right).$$

When N is finite, the individual's plan is partially time-inconsistent. While consumption within the individual's lifetime, $0 \leqslant t \leqslant 1$, and the value of K_1 are time-consistent, each descendant ($n > 0$), if he were to replicate the individual's measure of optimism, N, would need to revise his father's plan by adding one more generation to his father's plan, thereby altering the consumption programme from the beginning of his lifetime onwards. However, when N tends to infinity, the individual's plan is time-consistent since the number of future generations in his planning horizon does not decline in the generational order.

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NOTES

1. The structure of a 'hybrid' model and a detailed derivation of the results are available from the authors upon request.
2. This can be seen most clearly from a rewrite of (2) as

$$W_n = (\alpha e^{-\delta})^n \int_{t=n}^{n+1} e^{-\delta(t-n)} u(c_t)\, dt.$$

3. Having assumed that $\alpha e^{-\delta} < 1$,

$$\sum_{n=0}^{N} \alpha^n e^{-\delta n}$$

is the sum of a geometric series. For a large N this sum tends from below to $(1 - \alpha e^{-\delta})^{-1} < 1$. Therefore, $H_1 = K_0[1 - 1/(1 - \alpha e^{\delta})^{-1}] = K_0 \alpha e^{-\delta}$ and hence, $0 < H_1 < K_0$; the present value of the consumption source that the individual optimally bequeathes is strictly positive.
4. From (19) we get that c_t positively relates to K_0, and from (16) we have that K_1 positively relates to K_0. Hence c_t positively relates to K_1.

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